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ABSTRACT

This guide is designed to assist inservice providers in conducting successful workshops for teachers, administrators, and others associated with Chapter 1 mathematics programs. It contains step-by-step procedures for preparing, organizing, and presenting the workshop. Included in this guide are: (1) an advanced planner, which includes a detailed checklist for materials and equipment needed to conduct a successful problem-solving workshops, and a workshop outline, which includes detailed instructions with the goals of the workshop, specific problem-solving activities, and recommendations for using the workshop materials; (2) blackline masters for the participant handouts and overhead transparencies; (3) a brief overview of problem-solving topics; (4) resource materials and support materials including additional problems and suggestions; and (5) a bibliography of 12 additional resources. (CW)

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Chapter 1
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Region B
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WORKSHOP LEADER'S GUIDE

Mathematics Problem Solving

A more advanced skill for Chapter 1

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SE051838

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1 hour Workshop

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What's in this Guide?

This Workshop Leader's Guide on Mathematics Problem Solving is designed to assist Technical Assistance Center staff members and other inservice providers in conducting successful workshops for teachers, administrators, and others associated with Chapter 1 mathematics programs. This guide contains step-by-step procedures for preparing, organizing, and presenting the Mathematics Problem-Solving Workshop.

Contents:

Section 1 includes the **Introduction**, **Getting Started**, and the **Workshop Outline**.

The **Getting Started** section includes an Advance Planner and a detailed checklist for materials and equipment needed to conduct a successful problem-solving workshop.

The **Workshop Outline** includes detailed instructions for presenting a one-hour workshop including the goals of the workshop, specific problem-solving activities, and recommendations for using the overhead transparencies and participant handouts . The outline for the one-hour workshop can be expanded to provide the basis for a workshop up to three or more hours in length.

Sections 2 and 3 contain the blackline masters for the participant handouts and overhead transparencies referred to in the **Workshop Outline**.

Section 4, the **Background** section, gives a brief overview of problem-solving topics with which you should become familiar.

Sections 5 and **6** contain **Resource Articles** and **Support Materials** including additional problems and suggestions for presenting a workshop on problem solving.

Section 7 includes a **Bibliography** of additional sources of information.

How to Use This Guide

This Guide contains the planning and presentation materials necessary to conduct a successful workshop on problem solving. The materials were developed to allow a great deal of flexibility. Suggestions for workshop variations and a variety of problems are included so that the workshop can be adjusted to fit the needs and mathematical backgrounds of your audience. A wide range of problem-solving approaches, types of problems, and specific research can be used in the actual workshop presentation. You may choose to change, add, or eliminate a problem, activity, or transparency. The sections in the Workshop Outline titled "Gaining an Understanding" and "A Model for Teaching Problem Solving" are well suited for expansion or contraction, depending upon your presentation needs.

Getting Started

Begin your advance planning for the workshop by establishing some of the initial details, such items as date, place, and type of audience. (See **Advance Planner,** a simple checklist for planning a workshop, p.4.) Then begin studying the contents of this guide by following the ***G-U-I-D-E*** steps outlined below: *Glance*, *Understand*, *Investigate*, *Develop*, and *Edit*.

G-U-I-D-E

- *Glance* through the entire set of materials. This will give you a feel for the types of materials contained in the Guide (and their location) when you study the details later.

- *Understand* as many of the materials contained in the Guide as possible. Plan enough time to develop a full grasp of the materials in order to make more informed decisions about your workshop presentation.

- *Investigate* further. You may want to do additional research, try different problems, or experiment with various activities.

- *Develop* additional materials. These may be workshop notes, transparencies, handout pages, activities, or any item resulting from your "investigating" activities.

- *Edit.* Look carefully at the total picture, then elaborate or eliminate, if necessary.

Begin planning as soon as possible. Even if you use only the materials in this Guide, the ***G-U-I-D-E*** steps will take time and should be included in your planning. It is especially important to allow yourself the opportunity to thoroughly explore and solve the problems in the **Workshop Outline** so the purpose and strategies for each problem will become more apparent to you. In addition, as you engage in the problem-solving experience, you are likely to discover that you need or want to try additional problems and activities from several sources. (See Section 6, **Support Materials**.) You may find that these materials are better suited to your particular workshop and, therefore, you may want to substitute them for other workshop materials in earlier sections of this guide.

What You Need for the Workshop:

EQUIPMENT

___ Overhead Transparency Projector
- ___ extension cord
- ___ 3-way plug adaptor
- ___ extra bulb or spare projector

____ Blank Overhead Transparencies

____ Screen

____ Microphone (if needed)

MATERIALS

___ Workshop Outline

____ Supporting Notes

____ Participant Handouts (one for each participant from Section 2)

____ Overhead Transparencies (prepared from masters in Section 3)

SUPPORT MATERIALS

____ Chart paper

____ Poster board

____ Marker and tape (or chalkboard and chalk)

Before You Begin

Make copies of the overhead transparencies you plan to use in the workshop, and be sure you have one copy of the handouts for each participant.. If you are presenting the workshop in a location with which you are unfamiliar, ask the local contact person to be sure the equipment listed above is available and in working order on the scheduled day and time of the workshop. If you will be supplying your own equipment, make arrangements for obtaining it well in advance of the workshop and make sure everything is in working order.

Workshop Advance Planner

Presentation Information

Title _____

Date _____ Day _____ Time _____

Place _____

_____

Audience Type _____ Number _____

Purpose _____

Contact Person _____ Phone _____

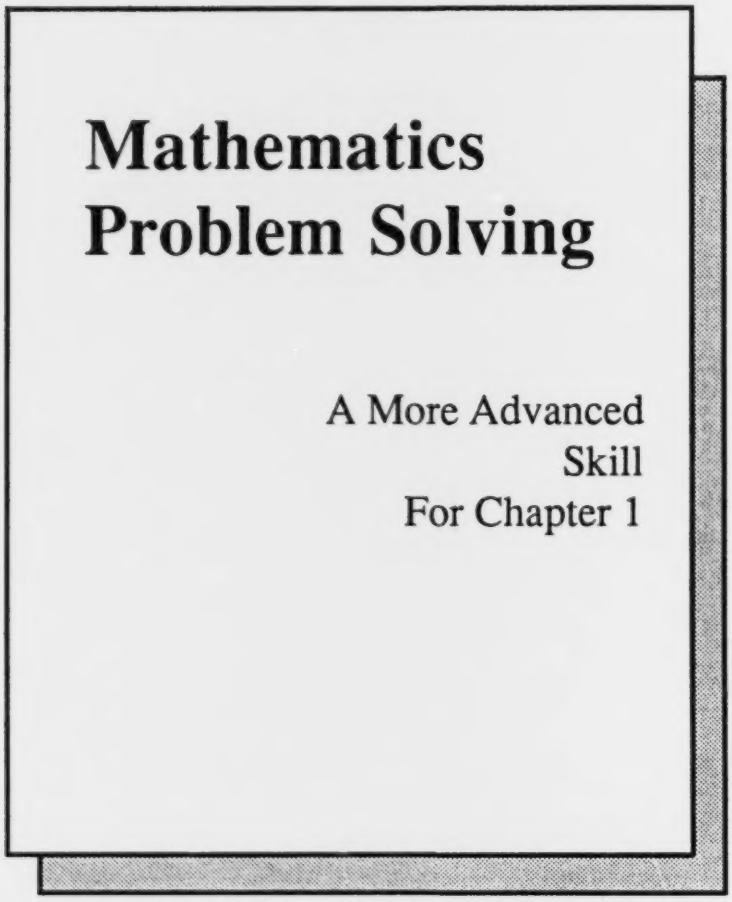
Planning Task	Date Completed
Contact Person(s) for Planning	
Confirm Date, Time & Place	
Make Travel and Hotel Plans	
Arrange for Equipment	
Send Workshop Agenda to Contact	
Personalize Workshop Outline	
Other _____	

MATHEMATICS PROBLEM SOLVING
One-Hour Workshop Outline

Setting the Tone	**10 minutes**
Goals and Background	**10 minutes**
Gaining an Understanding	**20 minutes**
A Model for Teaching Problem Solving	**10 minutes**
Summary and Conclusion	**10 minutes**

Setting the Tone

(10 minutes)



Display T-1: Workshop Title Slide: "Mathematics Problem Solving"

Activity 1

If a group of six participants were to introduce themselves to one another, how many handshakes would it take

(*It would take 15 handshakes*)

Read Activity 1: "Shaking Hands"

Introduce yourself and describe your association with Chapter 1. Then follow the procedure below for Activity 1 which involves participants meeting one another.

Procedure:

Encourage participants to work on the problems in groups, sharing methods, answers, etc. Groups should write their answers on paper, not call them out. Rather than discuss only the correct answer(s), participants should discuss strategies they used to arrive at their answers. While the groups are working, circulate around the room looking for different strategies.

After a minute or two, announce that 30 and 36 are <u>not</u> correct solutions. This will encourage those who "jumped" to incorrect solutions to continue working toward a correct solution.

In the total group, discuss different strategies used by participants. By observing various groups, you will see some very different approaches.

Use strategies shared by participants to start a "Problem Solving Strategies" list. Use chalkboard, chart paper, or poster board to model the process for participants.

Strategies will be added to this list throughout the workshop. Please refer to Background Paper (Section 4) for further information about problem-solving strategies. Strategies used for the handshake problem might include:

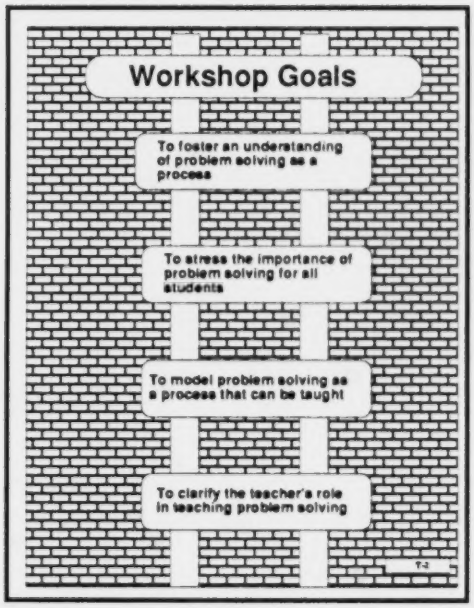
- draw a picture
- make a chart
- act it out

The list should develop from discussion and may include different names like "make a sketch" instead of "draw a picture" or may have more or fewer strategies.

Extend the activity above by asking "What if...?" questions such as "What if there were 10, 20, or 100 people in the group?" You might also discuss ways to adapt the problem to younger children, fewer children, or a role-playing situation.

Goals and Background

(10 minutes)


Workshop Goals
To foster an understanding of problem solving as a process
To stress the importance of problem solving for all students
To model problem solving as a process that can be taught
To clarify the teacher's role in teaching problem solving
T-2

(These goals are also listed on Participant Handout, H-1.)

Display T-2: "Workshop Goals"

Present the goals of the workshop. Briefly describe how you plan to achieve these goals during the workshop.

Experiencing the workshop:

Encourage participants to think about their workshop experiences in three ways, as a:

Participant

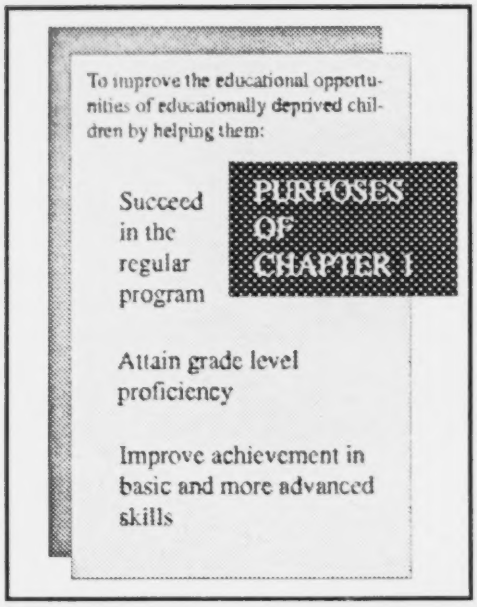
experiencing the feeling of satisfaction during problem solving

Teacher

learning about the process of problem solving, choosing good problems, and modeling problem-solving behavior

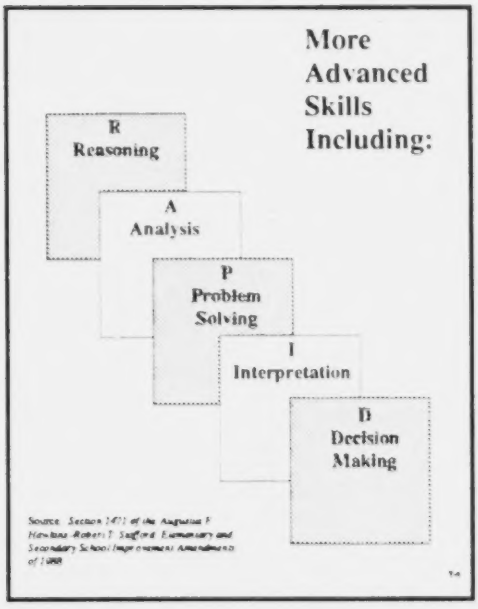
Student

developing an awareness of problem-solving strategies and the feeling of success that accompanies it


To improve the educational opportunities of educationally deprived children by helping them:
Succeed in the regular program
PURPOSES OF CHAPTER 1
Attain grade level proficiency
Improve achievement in basic and more advanced skills

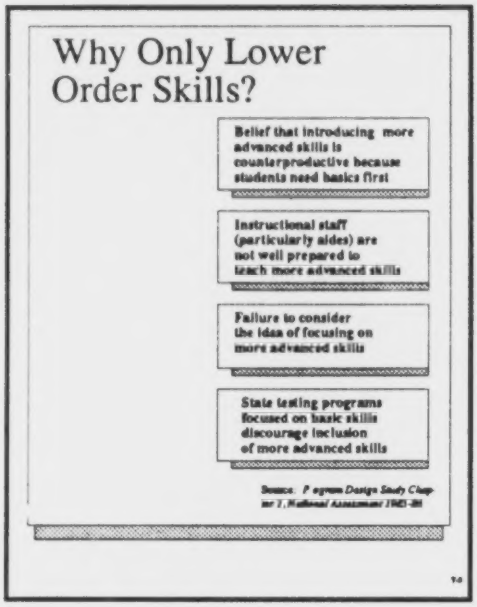
Display T-3:"Purposes of Chapter 1"

Discuss the purposes of Chapter 1 and the emphasis on more advanced skills in the Chapter 1 law.



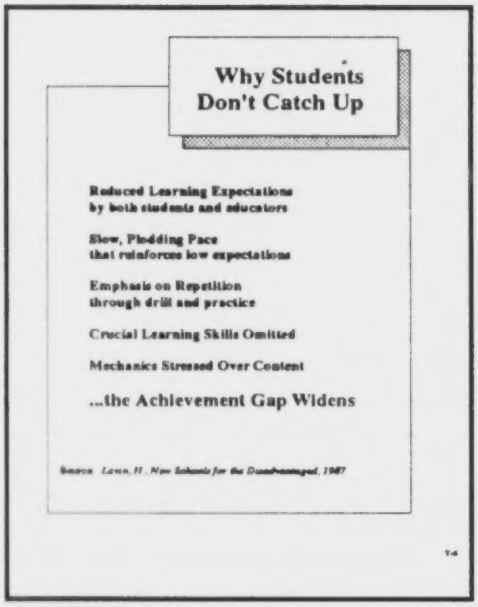
Display T-4:"More Advanced Skills"

Point out the acronym RAPID. Mention that other skills such as organizing, integrating, and evaluating may be included as more advanced skills.


Why Only Lower Order Skills?
Belief that introducing more advanced skills is counterproductive because students need basics first
Instructional staff (particularly aides) are not well prepared to teach more advanced skills
Failure to consider the idea of focusing on more advanced skills
State testing programs focused on basic skills discourage inclusion of more advanced skills

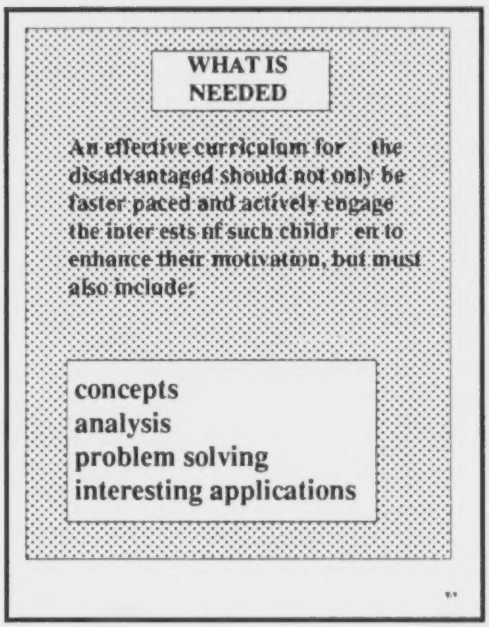
Display T-5: "Why Only Lower Order Skills?"

Discuss reasons for the traditional emphasis on lower order skills.


Why Students Don't Catch Up
Reduced Learning Expectations by both students and educators
Slow, Plodding Pace that reinforces low expectations
Emphasis on Repetition through drill and practice
Crucial Learning Skills Omitted
Mechanics Stressed Over Content
...the Achievement Gap Widens

Display T-6: "Why Students Don't Catch Up"

Discuss some of the reasons students don't "catch up" and suggestions for changing the situation. Mention items such as the use of an almost endless number of worksheets for computational drill and practice and the focus on mechanics or algorithms to the exclusion of concepts and problem solving.


WHAT IS NEEDED
An effective curriculum for the disadvantaged should not only be faster paced and actively engage the interests of such children to enhance their motivation, but must also include:
concepts
analysis
problem solving
interesting applications

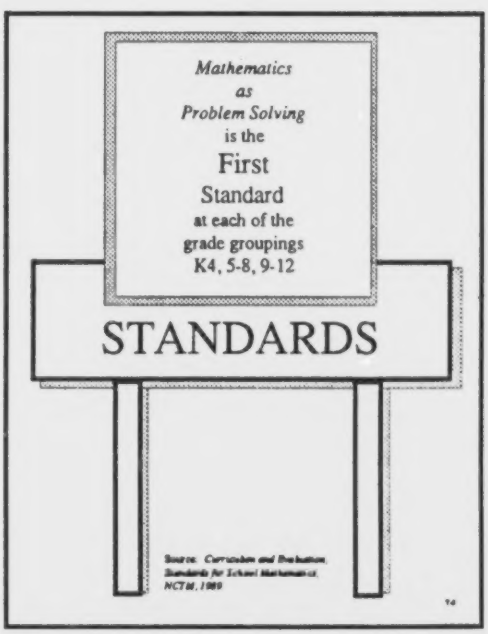
Display T-7: "What is Needed"

Discuss what changes may help in providing disadvantaged children with a more effective curriculum that would help them succeed at higher-order tasks when given the opportunity.

Display T-8: "NCTM Standards"

Discuss the importance of problem solving in today's mathematics curriculum, i.e., it is the *first* curriculum standard in each grade-level grouping.

Point out that problem solving is the reason for studying mathematics and that, as such, improved student problem-solving ability must be a goal for all mathematics instruction.



Gaining An Understanding of the Problem-Solving Process

(20 minutes)

Activity 2

Find the number of combinations of coins that could be used to buy a can of soft drink for 50¢ from a vending machine. The machine does not give change and does not take fifty-cent pieces or dollar bills.

(There are 10 possible combinations.)

Read Activity 2: "Making Change"

Present the problem in Activity 2; then follow the suggested procedure below.

Procedure:

A good way to "involve" students in this problem-solving activity is to present the problem as a story or relate it to a real-life situation. You may want to model this with a story similar to the following:

(Using a fifty cent piece as a prop for this problem may add interest to the story and prompt a discussion of why it is an unpopular coin.)

I was standing in front of a vending machine that dispensed 50¢ soft drinks. A woman came running up to me waving a large coin and acting very disturbed. When I asked if I could help, she told me she desperately needed a soft drink, but had only a fifty-cent piece. I told her I would give her change for the vending machine if she could tell me how many possible combinations of coins would work in the machine.

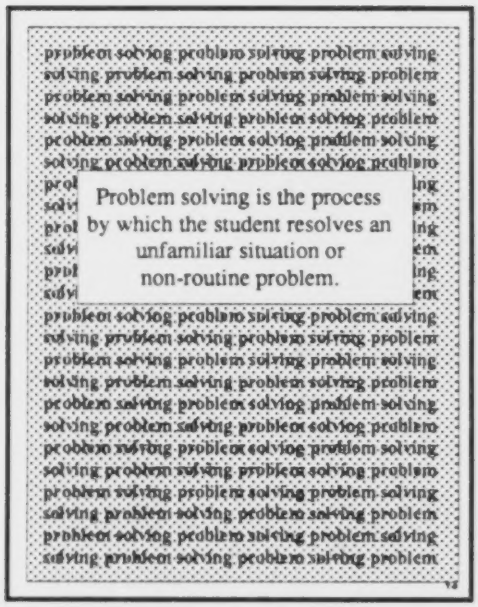
Again, encourage participants to work in groups, sharing strategies and writing their answers. Circulate, answering questions and looking for different strategies used. As a group, discuss strategies and add them to the list.

Possible strategies include:

- make a list
- make an organized list (a refinement of the first)
- act it out
- use real or play money
- use manipulatives

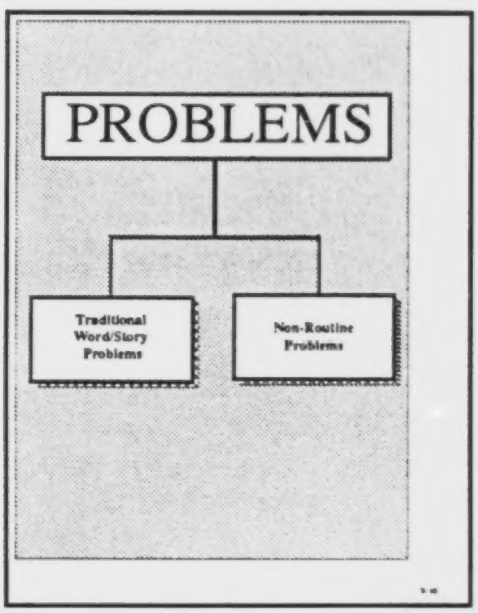
Extend the problem by asking "what if" questions, like, "What if we could use pennies?" or "What if the machine charged 60¢?" Discuss adapting the problem to younger children, perhaps having them use play or real coins.

Definition



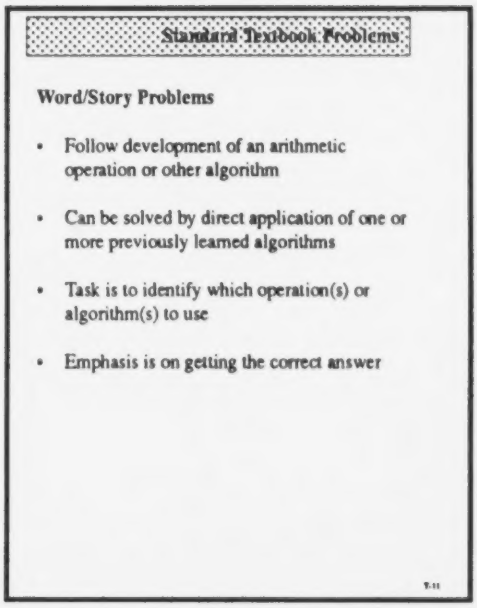
Display T-9: "Problem Solving"

Define problem solving. Emphasize that it is a process. Refer to the previously solved problems. Discuss what was involved in the solution. This discussion will help participants to better identify with and understand Polya's 4-step method.



Display T-10: "Problems"

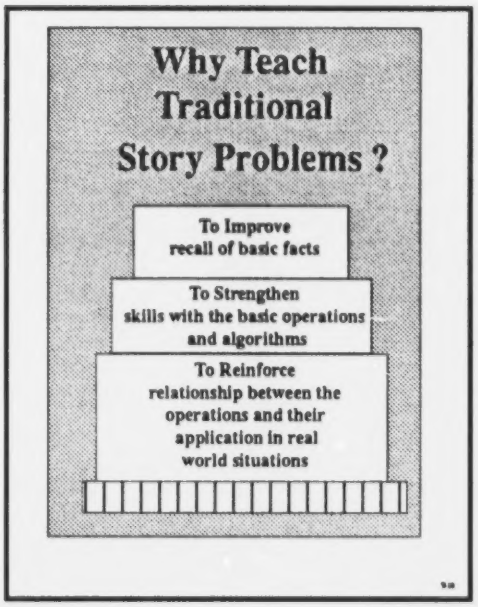
Explain that students need experience in solving all types of problems, including both the traditional word problems typically found in textbooks and non-routine or process problems.


Standard Textbook Problems
Word/Story Problems
• Follow development of an arithmetic operation or other algorithm
• Can be solved by direct application of one or more previously learned algorithms
• Task is to identify which operation(s) or algorithm(s) to use
• Emphasis is on getting the correct answer

Display T-11: "Standard Textbook Problems"

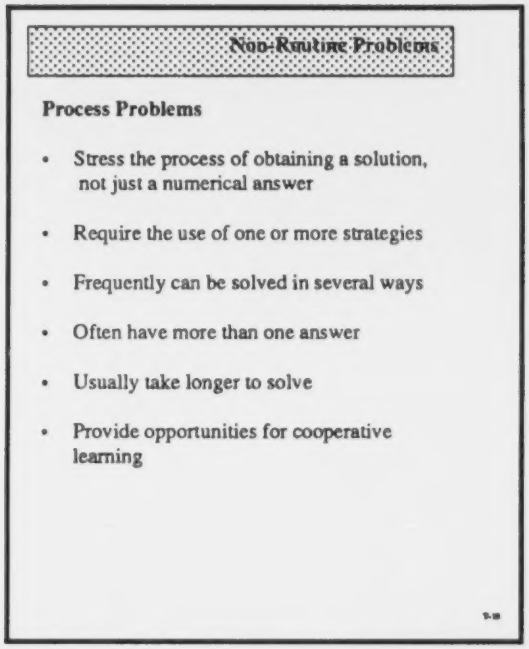
Ask participants where story problems are usually found in math textbooks. (Most often, they are at the beginning or end of a page or chapter.) Identify the attributes of standard textbook problems listed on the transparency. This should help bring to life many of the attributes of standard textbook problems.

(Note: An algorithm is a procedure that frequently involves repetition of an operation or a series of steps. One example is the long division algorithm.)


Why Teach Traditional Story Problems ?
To Improve recall of basic facts
To Strengthen skills with the basic operations and algorithms
To Reinforce relationship between the operations and their application in real world situations

Display T-12: "Why Teach Traditional Story Problems"

Point out that textbook problems *are* important. They can help improve the recall of basic facts, strengthen skills with the basic operations and algorithms, and reinforce relationships between the operations and their application in real world situations.


Non-Routine Problems
Process Problems
• Stress the process of obtaining a solution, not just a numerical answer
• Require the use of one or more strategies
• Frequently can be solved in several ways
• Often have more than one answer
• Usually take longer to solve
• Provide opportunities for cooperative learning

Display T-13: "Non-routine Problems"

Review the six points on the transparency in light of participant involvement with the two problems presented in the workshop. Point out that most of the problems in the handout are considered "process problems".

Example:
Ask participants if the following question is a traditional story problem or a process problem:

There are 32 turkeys that are to be shipped in crates that hold 4 turkeys each. How many crates will be needed?

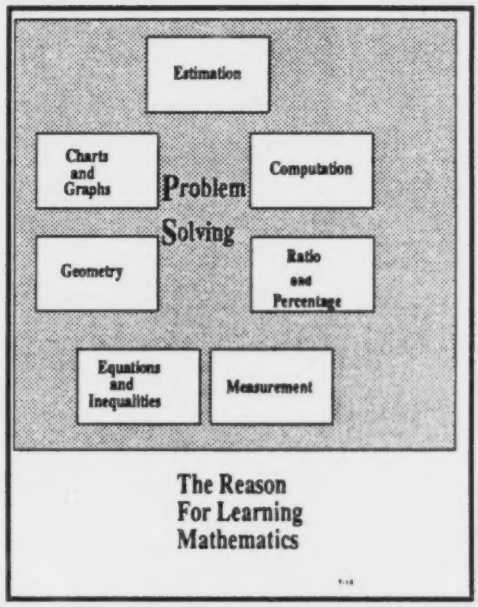
(The problem is from a fourth-grade mathematics text.)

Discuss whether this question would involve fourth-grade students in real problem solving. Compare this to the other problems in the workshop. Remind participants that problems of this type are important, but that students need experience in solving process problems. Process problems will improve their ability to solve both non-routine and textbook type problems.

Display T-14: "Problem Solving- The Reason for Learning Mathematics"

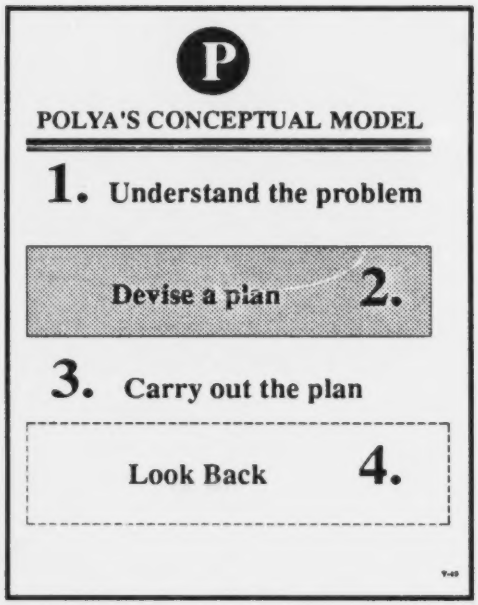
Stress that problem solving should be included throughout the mathematics curriculum. Problem solving should not be taught as a separate unit, or only at "special times," then forgotten. Teachers need to make problem solving the focus of mathematics instruction and take advantage of the many problem-solving opportunities that arise in the classroom.

Mention that additional value is placed on other skills such as computation and measurement when students realize the value of these skills for solving problems.



A Model For Teaching Problem Solving

(10 minutes)



Display T-15: "Polya's Model for Problem Solving"

Present the problem-solving model from George Polya's landmark 1945 book, **How to Solve It.** Mention that most current models of problem solving are adapted from this heuristic (or guide) for the problem-solving process originally proposed by Polya, who is often referred to as the "father of modern problem solving." Follow the procedure below to expand upon Polya's model. (See Handout H-11.)

Procedure:

Discuss the four steps of the problem-solving process in relation to the problems participants worked on during the workshop. Emphasize that learning to use a systematic, problem-solving process takes time, but once the process is discovered, it can be used over and over again. Encourage participants to *model* the four steps in their teaching rather than have students merely memorize the steps.

Consider displaying the model on a large piece of paper or poster board and taping it to the wall by the strategies list. Encourage participants to display the model permanently in the classroom for easy reference and use by the teachers and students.

Activity 3

A farmer has some pigs and chickens. He sent his son and daughter to count how many of each he had. The son counted 70 heads and the daughter counted 200 legs. How many of each does the farmer have?

(Ans.: 30 pigs and 40 chickens)

Read Activity 3:
"Pigs and Chickens"

Participants will again work in groups. Many times groups try to solve the problem (Step 3: Carry out the plan) before they thoroughly understand the problem or carefully devise a plan. As you circulate, suggest to groups having difficulty to use the steps in Polya's model.

Procedure:
"Trial and error" or "guess and check" is the most common strategy for solving this problem. Participants may feel that setting up algebraic equations is the only legitimate way to solve the problem. Note that trial and error is a legitimate problem-solving strategy and is commonly used in "real-world" mathematics. Point out that more than correct answers should be valued in the classroom. In this problem, the "mistakes" are not only useful, but essential to successfully solving the problem.

Point out that the strategies list is a valuable tool for problem solving in the classroom.

Teachers can:

- Create a strategies list with their students
  (add to the list as modeled in the workshop)
- Display the list openly for easy reference
- Encourage students to create their own strategies when needed
- Refer students to the list of strategies when "Devising a Plan"
  (Step 2 of Polya's Model)

Note: The One-Hour Workshop allows participants to use a variety of strategies but does not give them experience with re-using those methods previously listed.

Summary/Conclusion

(10 minutes)

Distribute the Participant Handouts and Review Their Contents:
Take a few minutes to review the contents of the handout. Much of the handout is not specifically covered in the workshop but is meant as a reference for future use by participants. Reviewing the handout content will encourage its later use.

H-1 Goals and Experiencing the Workshop

Review the workshop goals and how each was demonstrated during the presentation.

H-2 through H-5 Reduced Copies of Transparencies

Remind participants that they have reduced copies of the transparencies, so they will not need to copy them when they are displayed.

H-6 through H-10 Effective Mathematics Teaching

Review and discuss findings from research, myths to overcome, typical problem-solving activities in the classroom, and procedures for encouraging more effective problem solving.

H-11 Polya's Problem-Solving Model

Review Polya's problem-solving heuristic and its use as demonstrated on H-12.

H-l3 through H-23

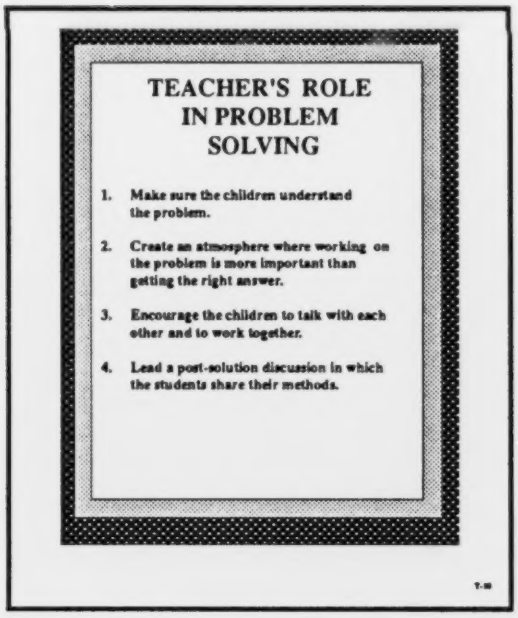
These pages contain a variety of process problems that can be used or adapted for Chapter l students. To promote the process of "how-to-solve" rather than that of "finding-the-answer," answers are deliberately not included.

H-13, *Visiting Friends,* is not easily adaptable for early elementary but is an excellent activity for participants. It can be included in a longer workshop format or substituted for one of the given activities in the one-hour workshop.

H-21 and **H-22** describe problem solving as used in cooperative groups. The problem example for older children, *Number Bracelets* , successfully models problem solving as a group activity and would be an excellent inclusion in a longer workshop.

H-23, *Sample Chapter l Desired Outcome...* includes a rating scale as well as an example of desired outcomes for problem solving.

H-24 gives a Bibliography listing of suggested readings. Remind participants that titles can be procured by requests on institutional letterhead.


TEACHER'S ROLE
IN PROBLEM
SOLVING
1. Make sure the children understand the problem.
2. Create an atmosphere where working on the problem is more important than getting the right answer.
3. Encourage the children to talk with each other and to work together.
4. Lead a post-solution discussion in which the students share their methods.

Display T-16:
"Teacher's Role in Problem Solving"

Remind participants that teachers need to *invite* their students to think. Research indicates that students retain knowledge more effectively when learning is experienced, when the student is actively involved . Thus the effectiveness of problem solving translates to knowledge and process retention.

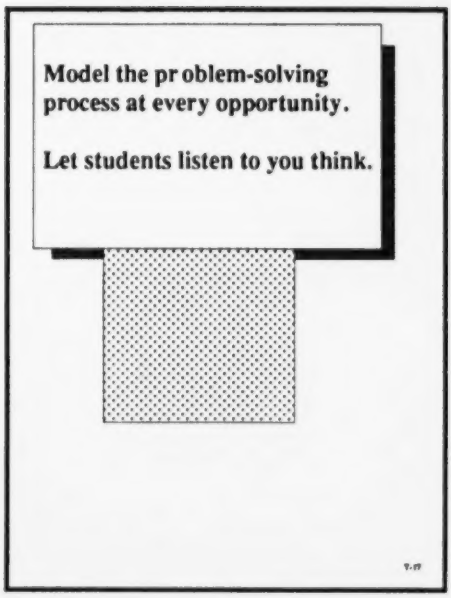
After going through the transparency, refer back to **H-6**, *Effective Mathematics Teaching (What Does Research Say).* Note that research findings indicate the success of utilizing problem solving in the classroom.

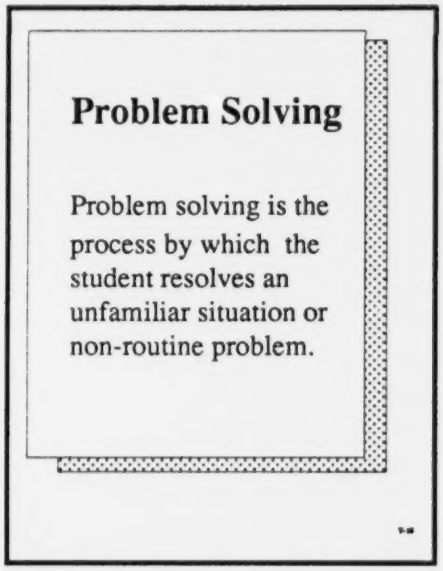
Refer to **H-7**, *Improving Problem Solving in Your Classroom*, and to **H-8,** *Self Assessment Questions for Teachers of Problem Solving*. These materials have been developed for participant use. **H-7** gives practical suggestions for expanding the use of problem solving in the particular teaching situation. **H-8** is a self-assessment tool designed to aid the individual in determining what teaching strategies they now effectively use and to indicate which strategies may need greater utilization.

Display T-17:

"Model the Problem-solving Process "

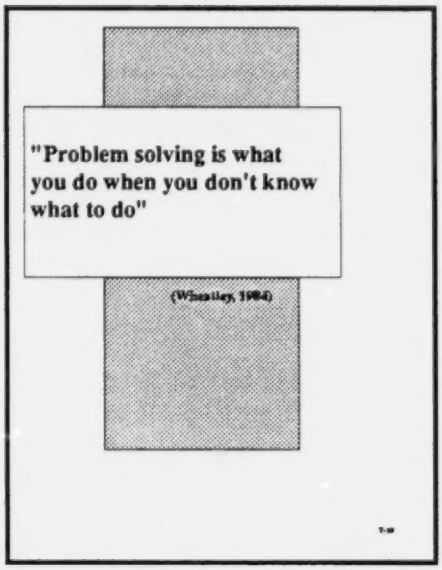
Teachers should be encouraged to *think aloud.* Such process modeling has gained popularity in the field, and research indicates that this instructional approach of letting students "in on the secrets" is an effective way of teaching not only problem solving, but reading comprehension and other advanced skills as well.




Problem Solving
Problem solving is the process by which the student resolves an unfamiliar situation or non-routine problem.

Display T-18: "Problem Solving"

Emphasize the term "process." Point out that teaching and utilizing problem solving isn't just a matter of equipping students with a bag of tricks but encompasses teaching students how to approach, analyze and understand a situation; devise a plan; implement a plan; assess the outcome; and adjust where necessary. Polya proposes strategies and a mind-set which empowers those who use them.



Display T-19: "Problem Solving is what you do..."

Read the statement emphatically. This quote encapsulates the complexity and difficulty of problem solving in a statement that is easily understood; it stresses for participants what problem solving should be for their students.

SUGGESTIONS FOR WORKSHOPS INVOLVING PROBLEM SOLVING

Leaders of workshops that involve problem solving should consider the following suggestions:

- Model the process of problem solving.
- Teach problem-solving heuristics -- drawing a picture, making a table, using trial and error, and so forth.
- Work the same problem several different ways, using different heuristics.
- Obtain an abundant supply of interesting problems.
- Have a variety of problems that are suited to the various mathematical backgrounds of the participants.
- Create a friendly, nonthreatening atmosphere.
- Have each participant work on problems that are individually challenging, but not overwhelming.
- Allot a large portion of the time for actually solving problems.
- Give participants opportunities to work cooperatively in small groups.
- Give participants ample opportunity to talk about solving problems.
- Emphasize estimation and reasonableness of results.
- Use calculators.
- Have participants generalize results.
- Encourage participants to create problems.
- Give participants a variety of nontraditional problems to take with them.
- Give participants a list of sources of good problems.

Source: *Professional Development for Teachers of Mathematics: A Handbook.* National Council of Teachers of Mathematics, National Council of Supervisors of Mathematics, 1986.

Mathematics Problem Solving

A More Advanced
Skill
For Chapter 1

Transparencies

Mathematics Problem Solving

A More Advanced
Skill
For Chapter 1

The One Hour Workshop

Workshop Goals

To foster an understanding of problem solving as a process

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T-2

PURPOSES OF CHAPTER 1

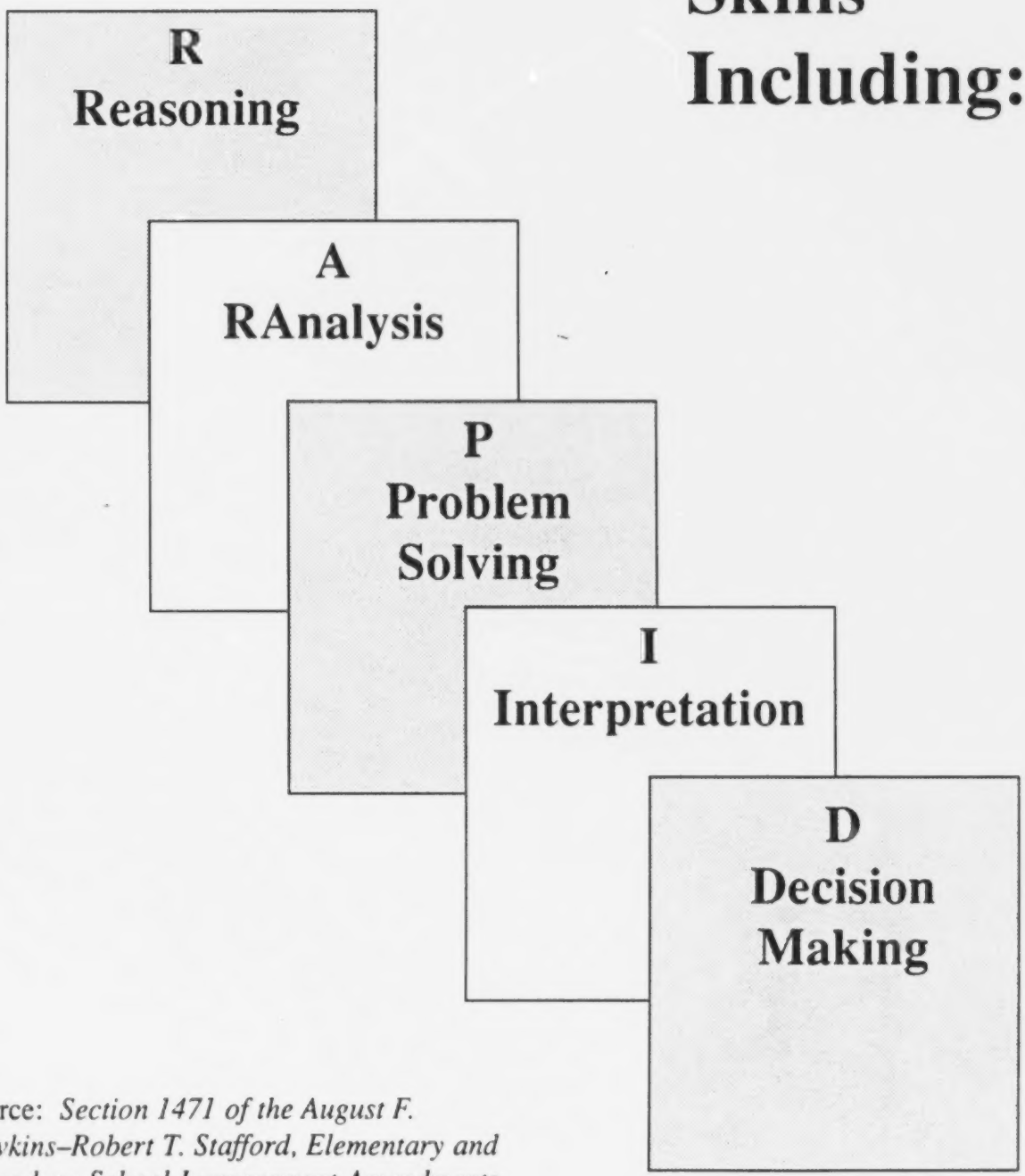
To improve the educational opportunities of educationally deprived children by helping them:

Succeed in the regular program

Attain grade level proficiency

Improve achievement in basic and more advanced skills

More Advanced Skills Including:



Source: *Section 1471 of the August F. Hawkins–Robert T. Stafford, Elementary and Secondary School Improvement Amendments of 1988*

Why Only Lower Order Skills?

Belief that introducing more advanced skills is counterproductive because students need basics first

Instructional staff (particularly aides) are not well prepared to teach more advanced skills

Failure to consider the idea of focusing on more advanced skills

State testing programs focused on basic skills discourage inclusion of more advanced skills

Source: *Program Design Study Chapter 1, National Assessment 1985-86*

Why Students Don't Catch Up

Reduced Learning Expectations by both students and educators

Slow, Plodding Pace that reinforces low expectations

Emphasis on Repetition through drill and practice

Crucial Learning Skills Omitted

Mechanics Stressed Over Content

...the Achievement Gap Widens

Source: *Levin, H., New Schools for the Disadvantaged, 1987*

WHAT IS NEEDED

An effective curriculum for the disadvantaged should not only be faster paced and actively engage the interests of such children to enhance their motivation, but must also include:

concepts
analysis
problem solving
interesting applications

Mathematics
as
Problem Solving
is the
First
Standard
at each of the
grade groupings
K4, 5-8, 9-12

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Source: *Curriculum and Evaluation, Standards for School Mathematics, NCTM, 1989*

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Standard Textbook Problems

Word/Story Problems

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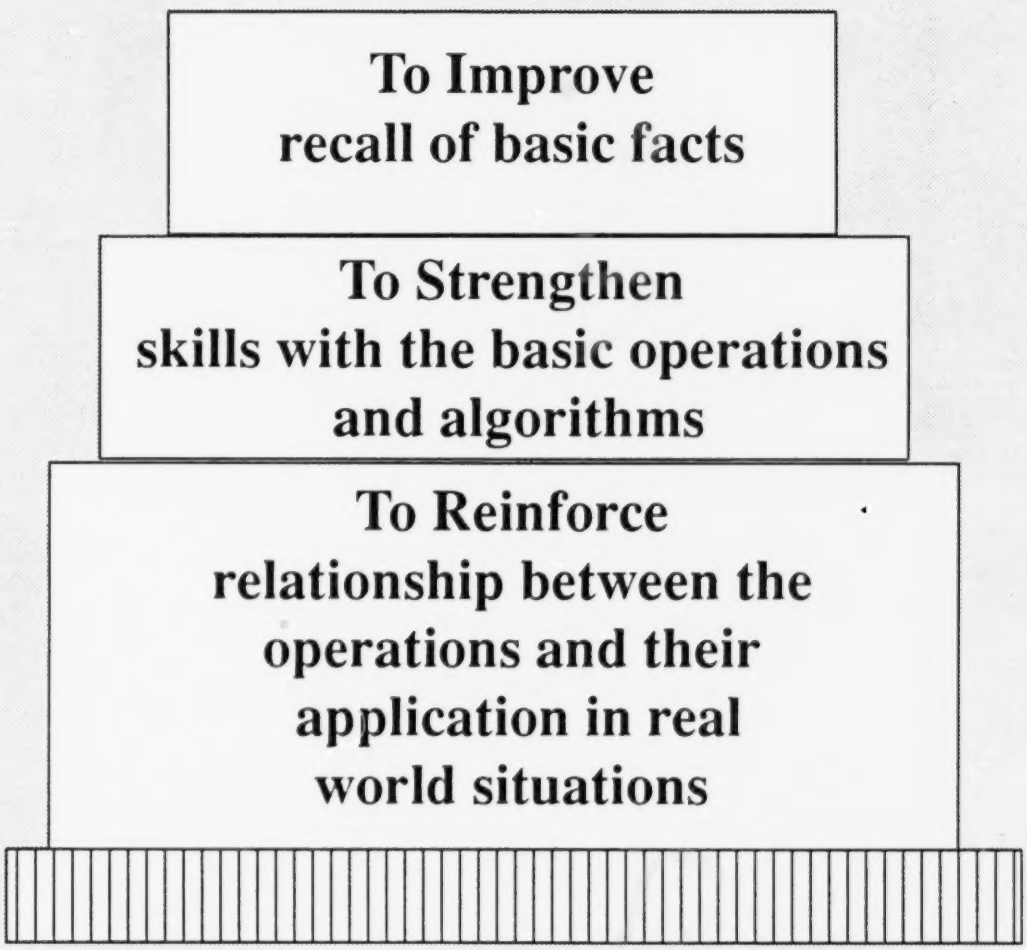
Why Teach Traditional Story Problems ?



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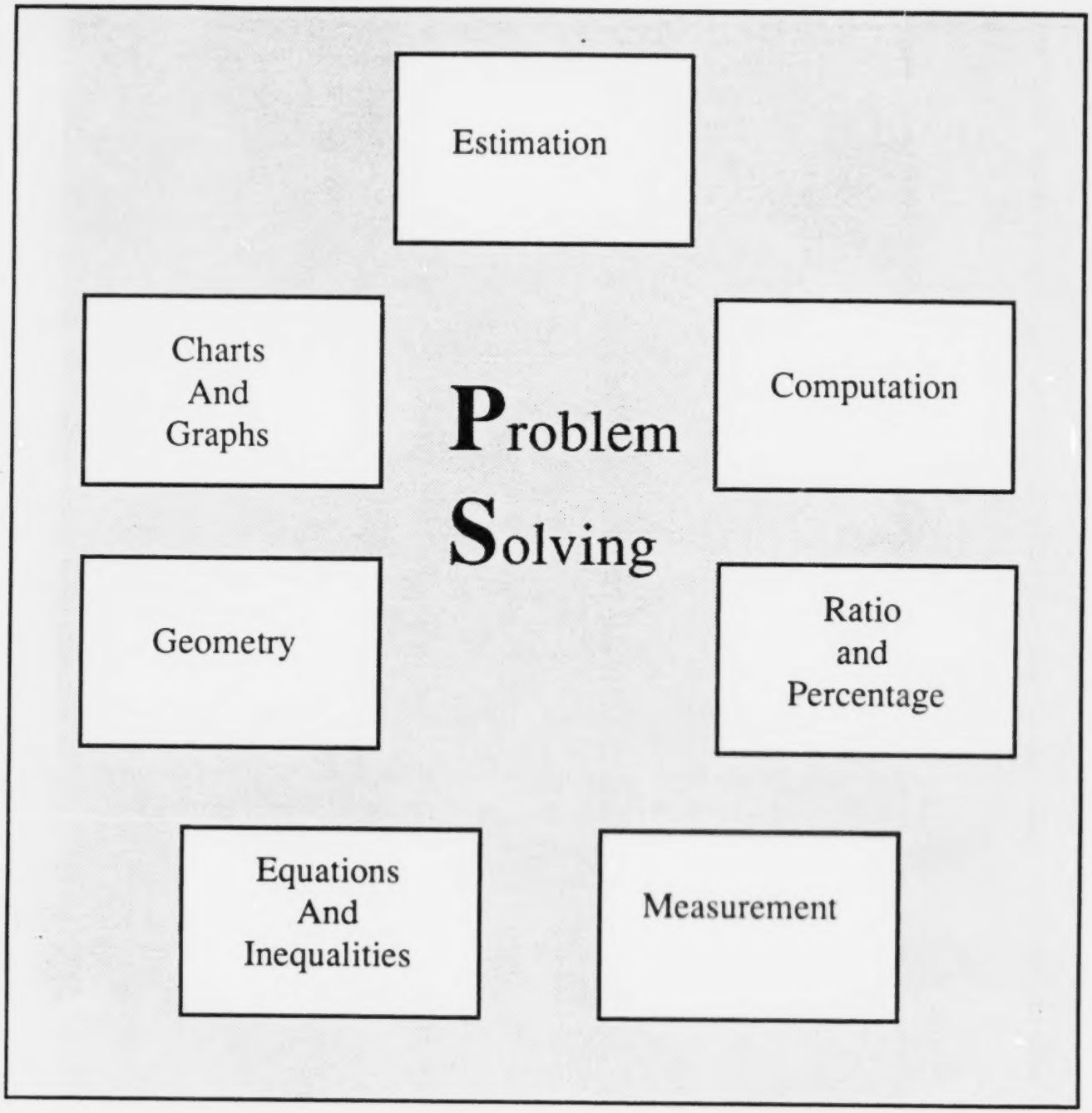

TheReason For Learning Mathematics

POLYA'S CONCEPTUAL MODEL

1. Understand the problem

Devise a plan 2.

3. Carry out the plan

Look Back 4.

TEACHER'S ROLE IN PROBLEM SOLVING

1. Make sure the children understand the problem.

2. Create an atmosphere where working on the problem is more important than getting the right answer.

3. Encourage the children to talk with each other and to work together.

4. Lead a post-solution discussion in which the students share their methods.

Model the problem-solving process at every opportunity.

Let students listen to you think.

Problem Solving

Problem solving is the process by which the student resolves an unfamiliar situation or non-routine problem.

"Problem solving is what you do when you don't know what to do"

(Wheatley, 1984)

Mathematics Problem Solving

A More Advanced
Skill
For Chapter 1

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Transparency 1

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Transparency 3

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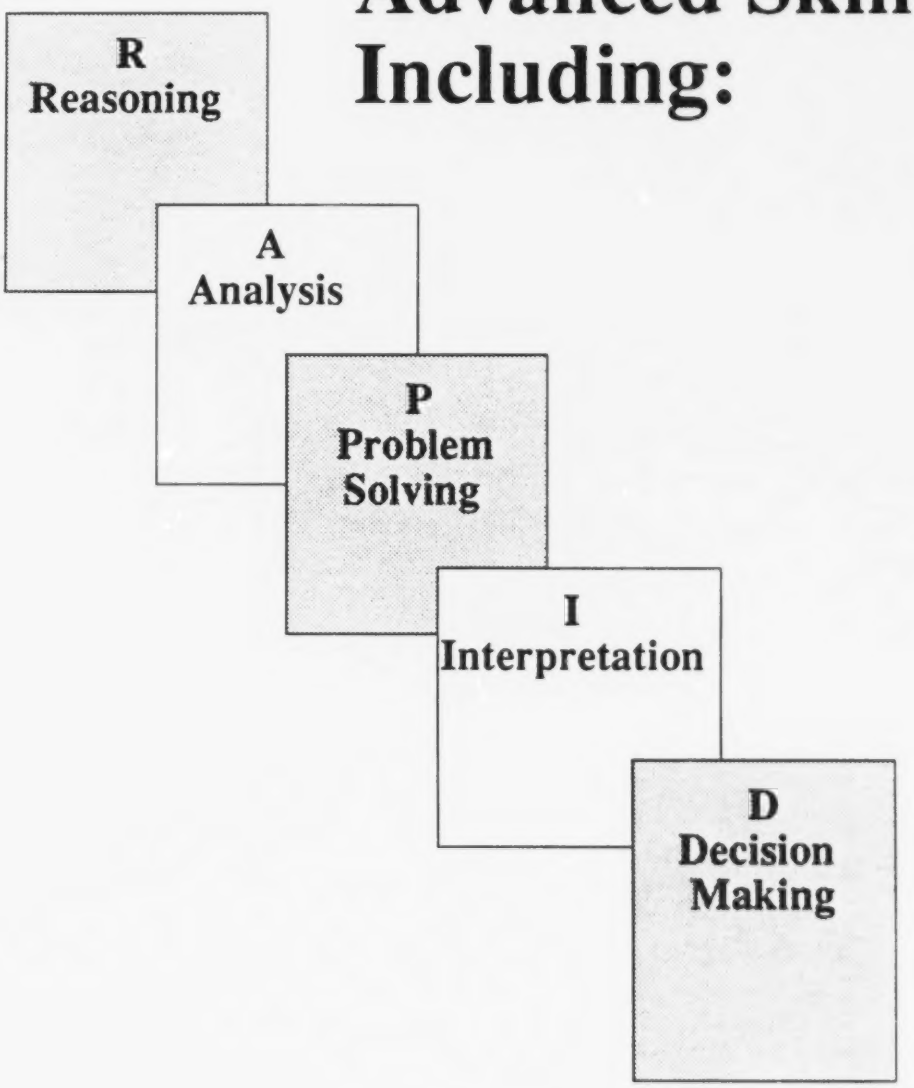

Source:
Section 1471 of the August F. Hawkins - Robert T. Stafford,
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Transparency 4

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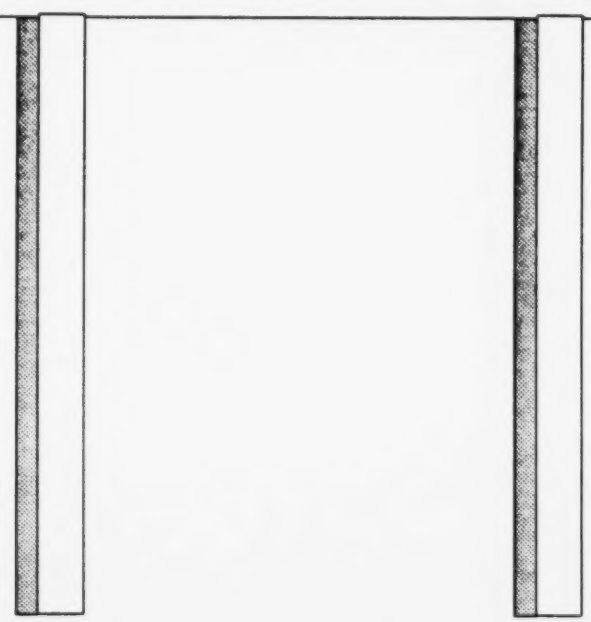

Transparency 8

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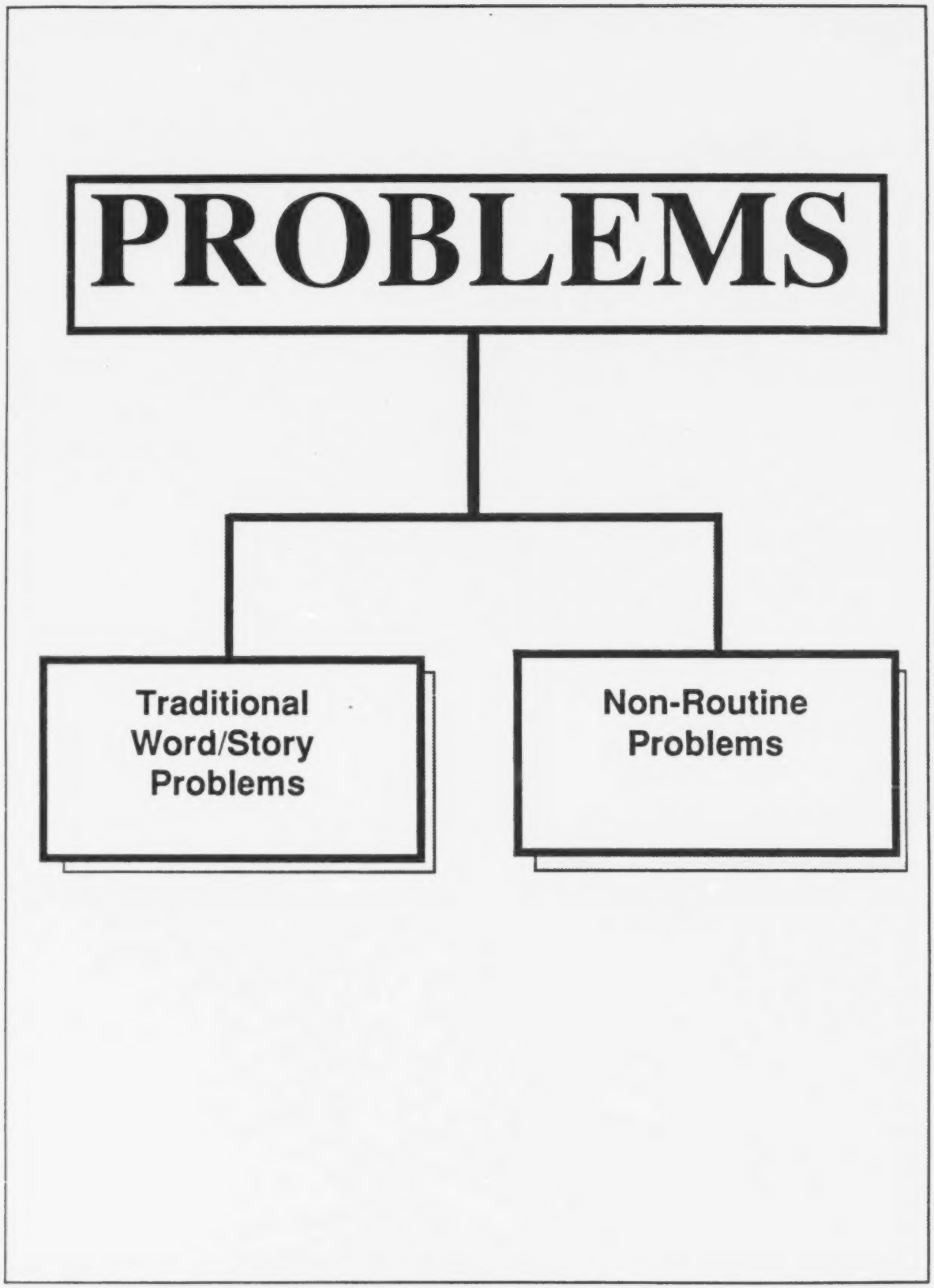

Transparency 10

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Transparency 11

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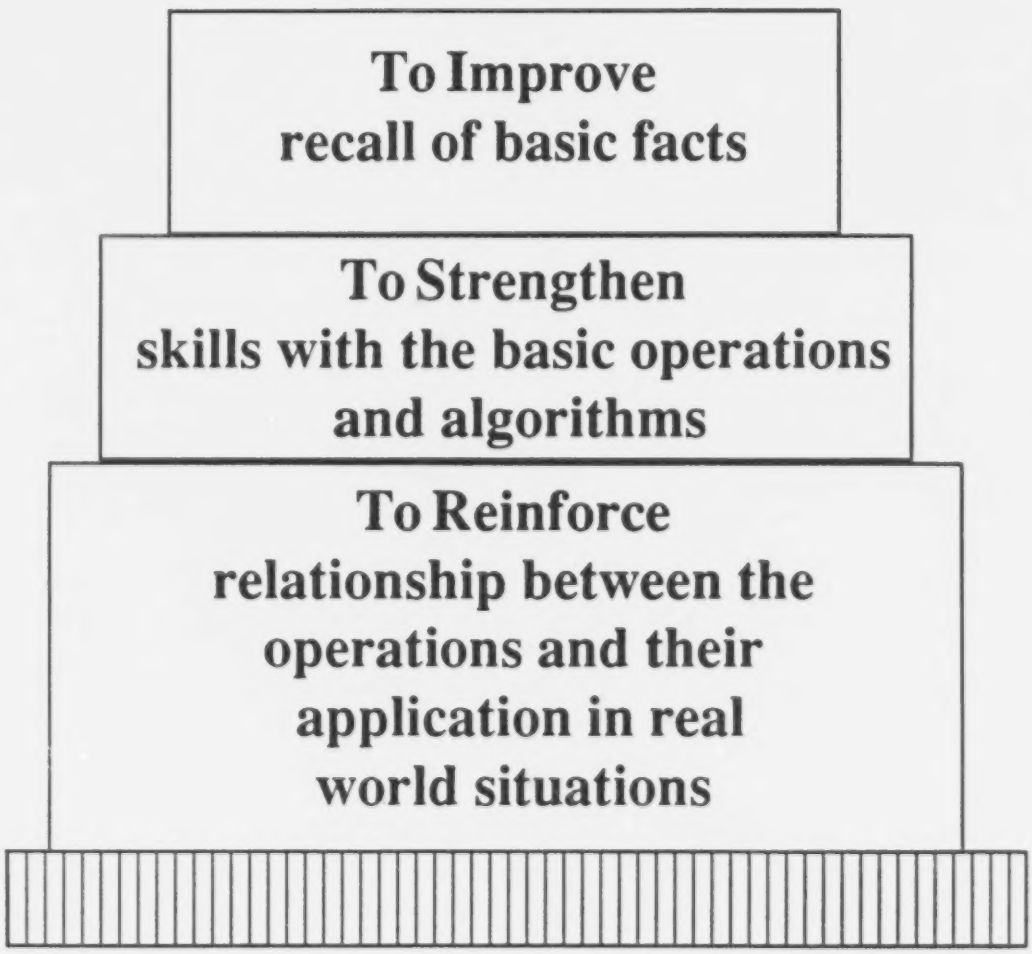

Transparency 12

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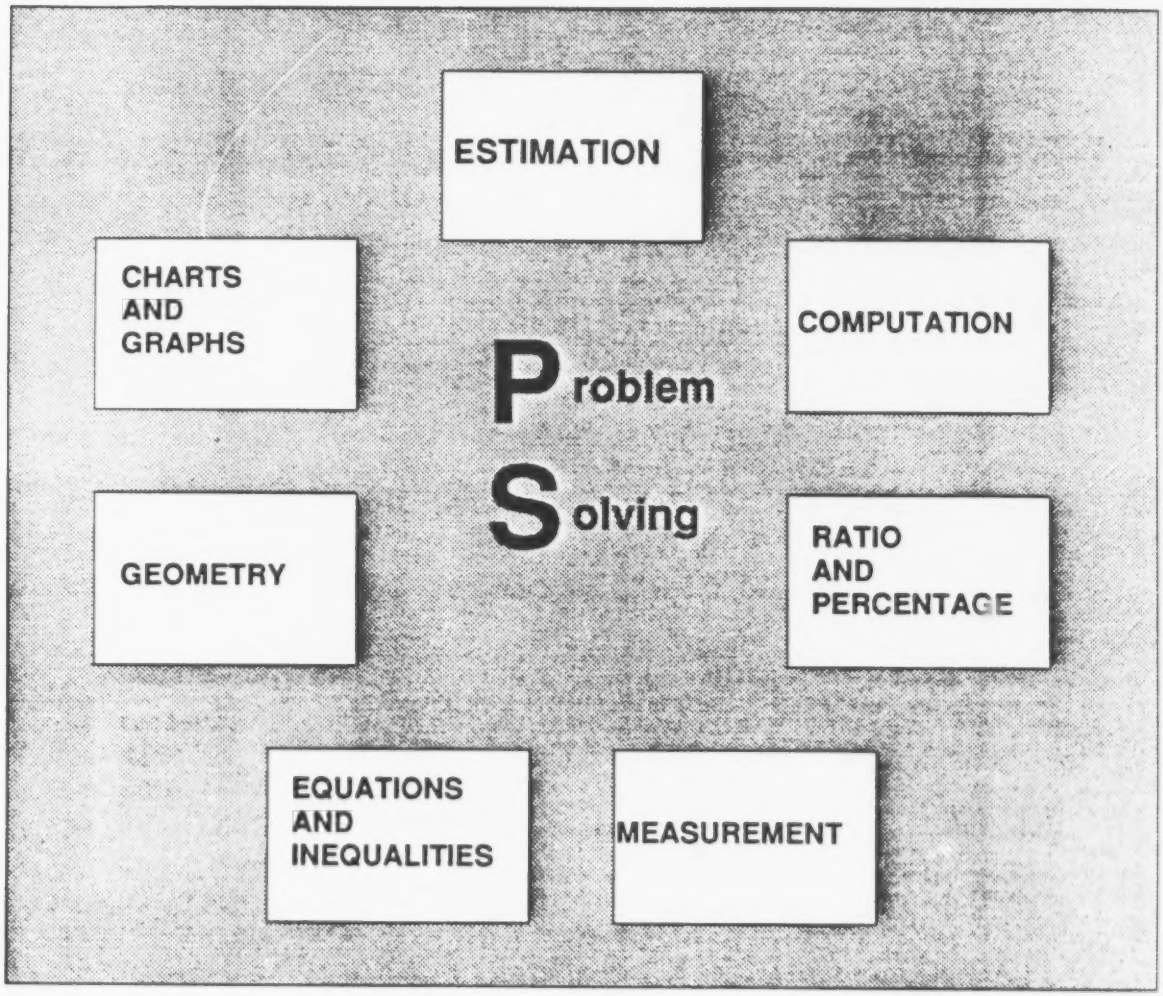

THE REASON FOR LEARNING MATHEMATICS

Transparency 14

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Transparency 15

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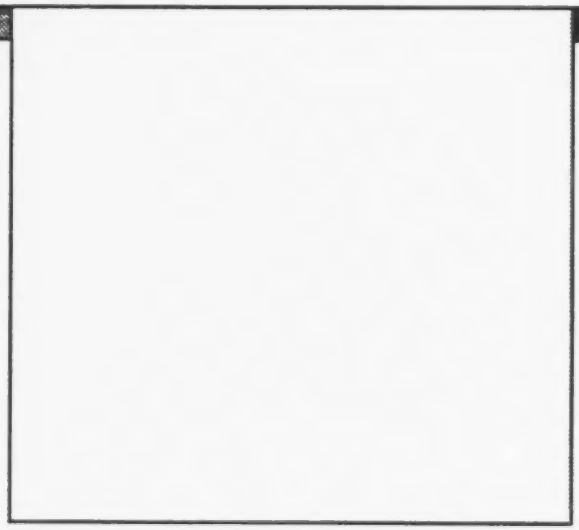

Tansparency 17

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Transparancy 18

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Transparancy 19

Alternate Activity 1

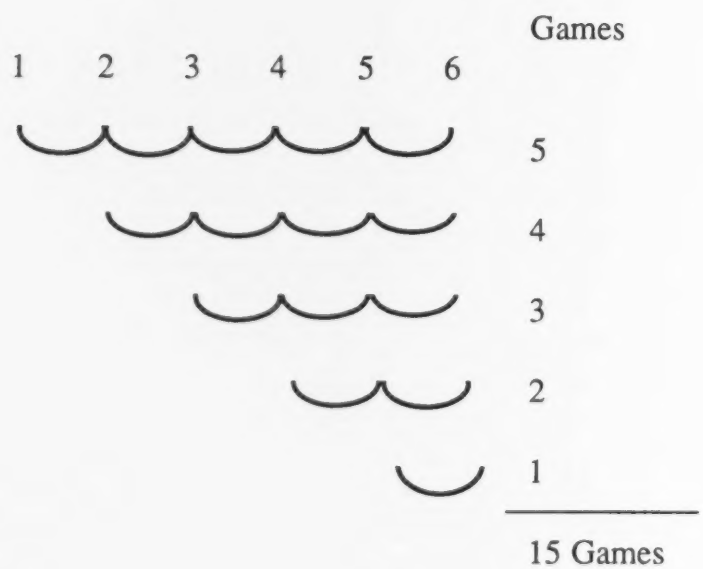
Six teams are in a junior vollyball league: The Spikers, Pounders, Setters, Servers, Netters, and the Vollys. If all six teams must play each other one time, how many games must be scheduled?

(It would take 15 Games)

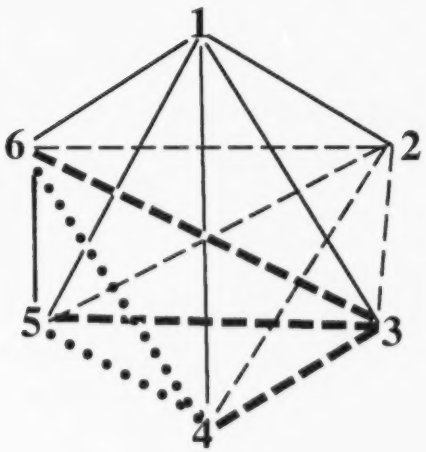
Alternate Activity 1

Possible Strategies:

1. Draw A Picture



2. Draw A Diagram



	Number of games
1 plays	5
2 plays	4 (Since already played 1)
3 plays	3 (Since already played 1& 2)
4 plays	2 (Since already played 1,2, &3)
5 plays	1 (Since already played 1,2,3,4)
	15 Games

3. Act It Out

People will assume a team name or number, then "play games" with each other until all have played each other once.

Alternate Activity 2

If you choose one food from each category on the menu, how many different combinations are possible?

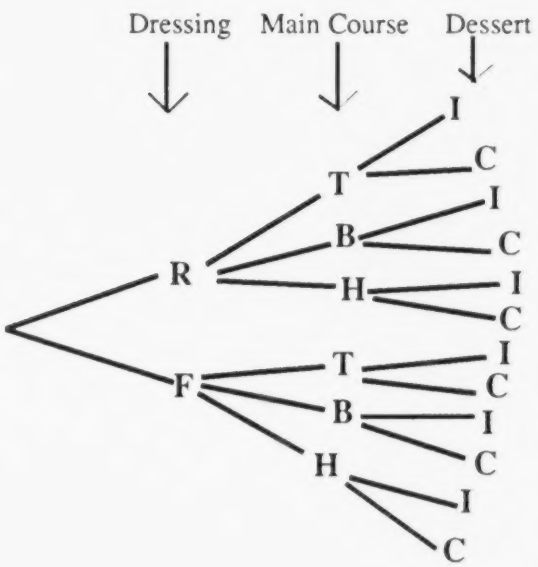
(12 combinations)

Alternate Activity 2

Salad	Main Course	Dessert
Ranch Dressing	Turkey	Ice Cream
French Dressing	Beef	Cake
	Ham	

Possible Strategies:

1. (Tree) Diagram



(12 combinations)

2. Organized List

Dressing	Main Course	Dessert
Ranch	Turkey	Ice Cream
Ranch	Turkey	Cake
Ranch	Beef	Ice Cream
Ranch	Beef	Cake
Ranch	Ham	Ice Cream
Ranch	Ham	Cake
French	Turkey	Ice Cream
French	Turkey	Cake
French	Beef	Ice Cream
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French	Ham	Ice Cream
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(12 combinations)

Alternate Activity 3

Alternate Activity 3

A toy manufacturer makes cars and motorcycles. The suppliers can only send 22 headlights and 44 tires. How many possible cars and motorcycles can be manufactured?
Extension: How many combinations are possible?

(There are 12 combinations)

Possible Strategies:

1. Create a Chart

Cars = 2 Headlights and 4 Tires
Motorcycles = 1 Headlight and 2 Tires

		Headlights	Tires
1	0 Cars	0	0
	22 Motorcycles	22	44
2	1 Car	2	4
	20 Motorcycles	20	40
3	2 Cars	4	8
	18 Motorcycles	18	36
4	3 Cars	6	12
	16 Motorcycles	16	32
5	4 Cars		
	14 Motorcycles		
6	5 Cars		
	14 Motorcycles		
7-12	Etc. (A total of 12 Combinations)		

Other Possible Strategies:

2. Guess & Check

3. Create an Oganized List

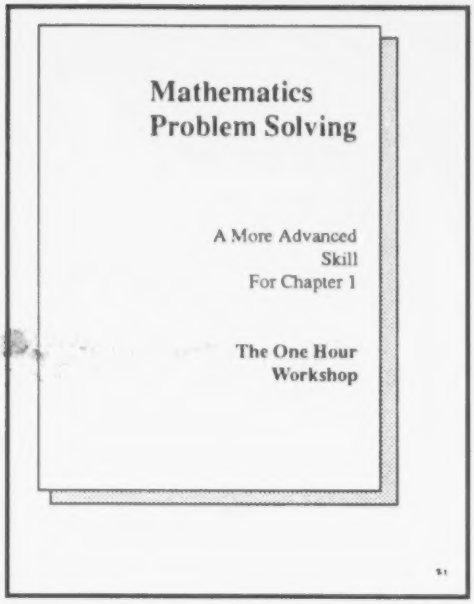
4. Find a Pattern

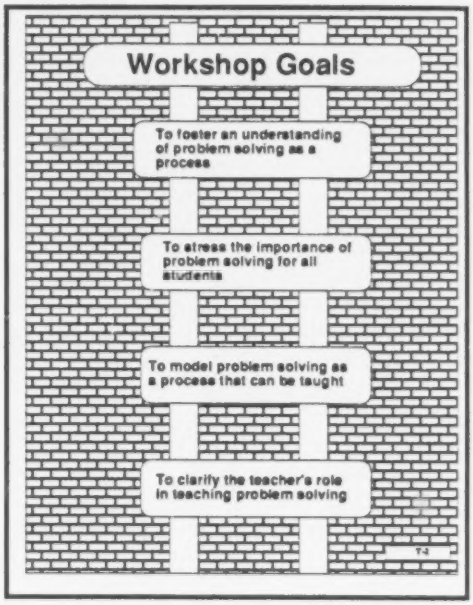
5. Algebraic:

$2\,C + 1\,M = 22$
$4\,C + 2\,M = 44$

Note: The algebra does not solve the problem in a simple step. Even algebra requires some additional work!

Slide Summary

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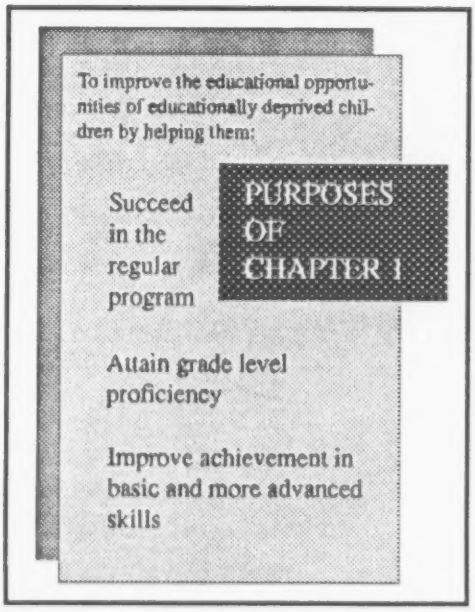
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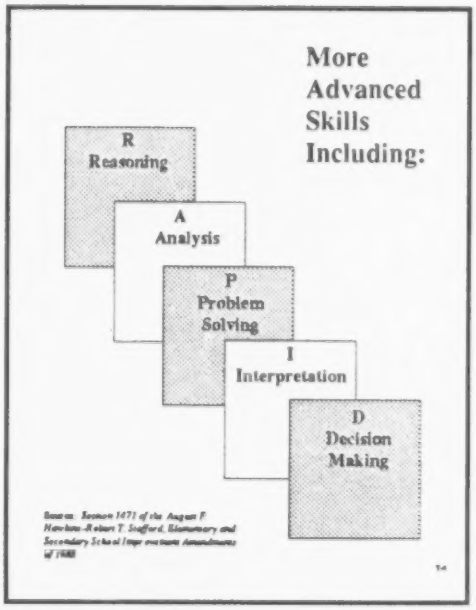

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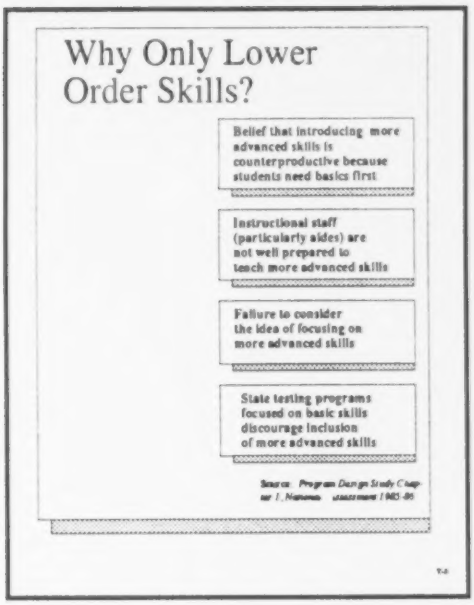
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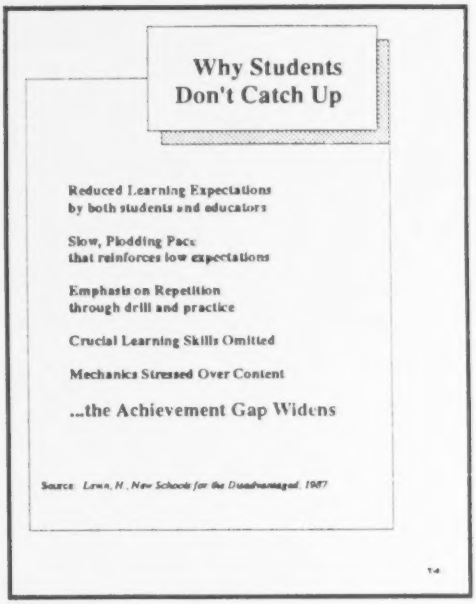
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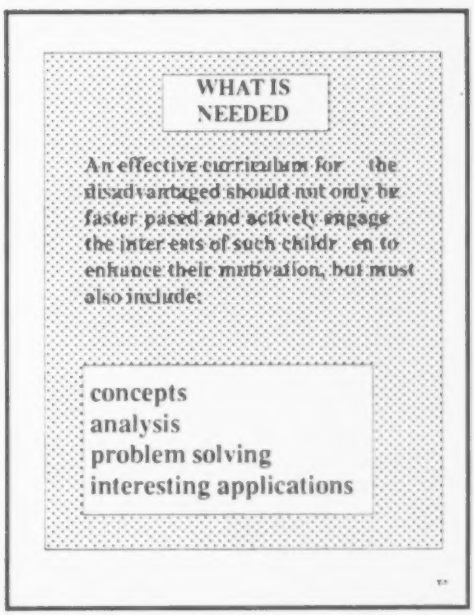
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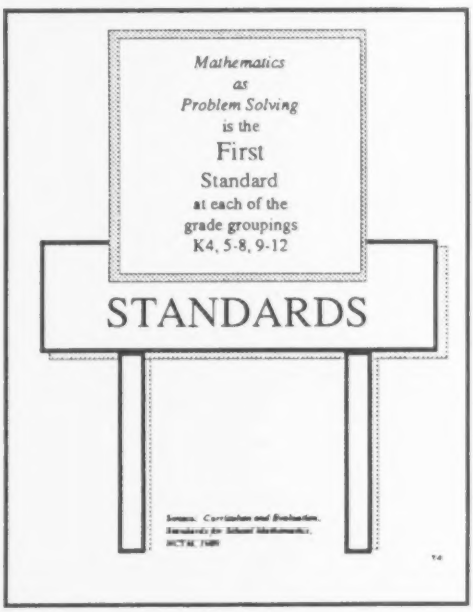
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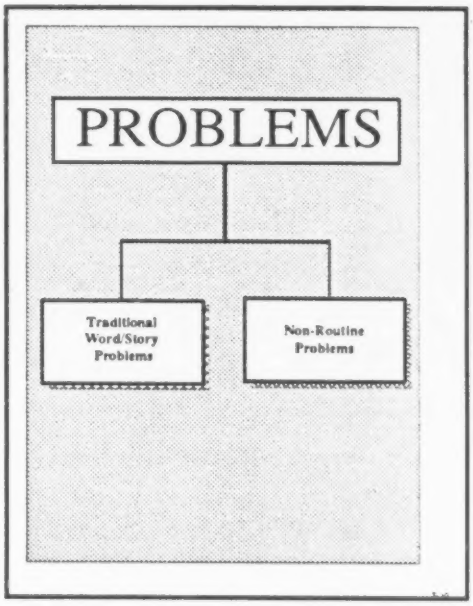

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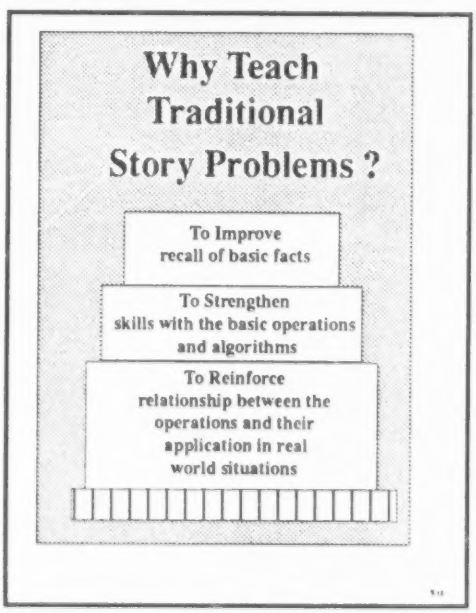
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To Reinforce relationship between the operations and their application in real world situations

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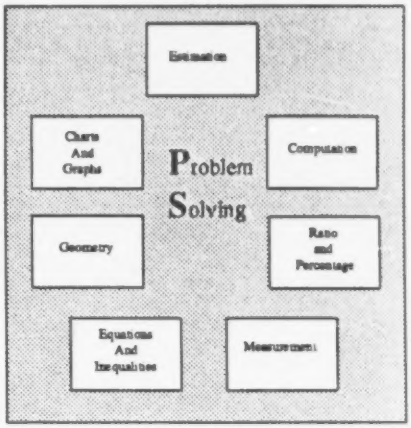

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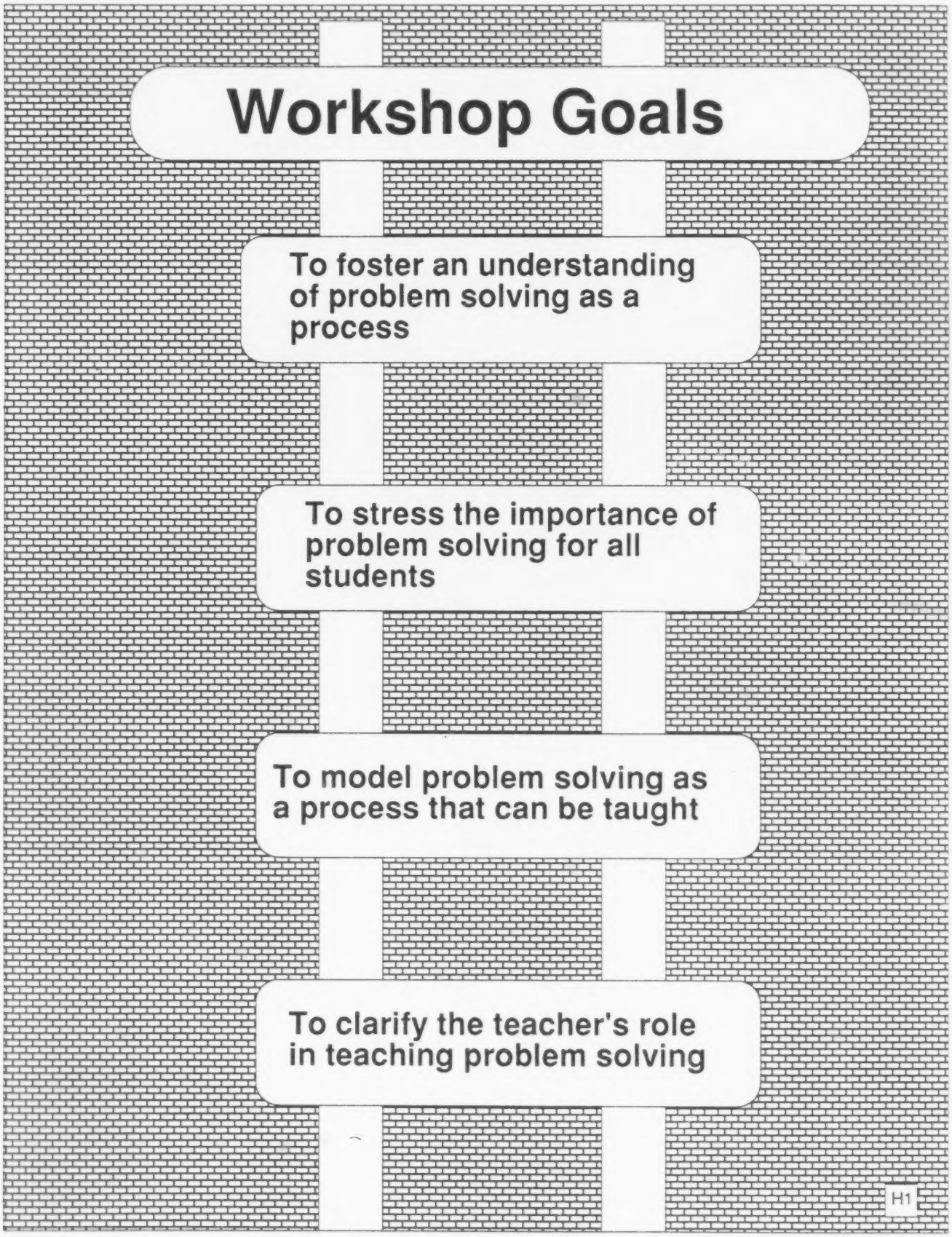
3. Encourage the children to talk with each other and to work together.
4. Lead a post-solution discussion in which the students share their methods.


Workshop Goals
To foster an understanding of problem solving as a process
To stress the importance of problem solving for all students
To model problem solving as a process that can be taught
To clarify the teacher's role in teaching problem solving
H1

EFFECTIVE MATHEMATICS TEACHING
(What Does Research Say?)

Class Organization:

The more effective teachers devote about half of each class to a combination of explaining, demonstrating, and discussing. The less effective teachers use only about a fourth of each class for these activities and over half of the class for individual seatwork.

Questioning:

The backbone for the teaching of problem solving is consistently asking questions that involve students and *invite them to think.*

Effective teachers ask more process questions (calling for explanations) and more product questions (calling for short answers) than do less effective teachers. They also ask more *new* questions after correct answers have been given.

Effective teachers also allow more wait-time between questions and answers — both the pause following a teacher's question and the pause following a student's response.

Encouragement:

Effective teachers try to keep most of their students, low achievers as well as high achievers, from slipping into passive learning. Effective teachers are more encouraging and receptive to student input than are their less effective colleagues. Effective teachers reinforce questions and requests for help.

Modeling:

More effective teachers engage in more problem-solving behavior. The teachers show the way through their own examples. Problem-solving instruction is most effective when students sense two things:

1. that the teacher regards problem solving as an important activity;

2. that the teacher actively engages in solving problems as part of mathematics instruction.

Source: *Research Within Reach: Secondary School Mathematics* (NCTM, 1982)

IMPROVING PROBLEM SOLVING IN YOUR CLASSROOM

MYTHS TO OVERCOME:

- We can't do that! These students don't know their basics yet!
- These students can't think abstractly.
- There is not enough time.
- If it is not in the textbook, it isn't important.

POSITIVE STEPS:

- Approach problem solving as a process, not as a series of specific skills or tricks to learn and apply.
- Select problems that interest students.
- Approach problems as "Here is a situation, let's think about it," rather than, "Here is a problem, let's find the answer."
- Take time with problem solving. Do not hurry the students into finding the answer.
- Act as a "facilitator" of problem solving. Careful modeling, questioning, and leading is more effective than telling.
- Encourage students to work together in problem-solving activities. Students should share their thoughts.
- Be patient. Be accepting of different approaches or methods. Encourage your students to tell *how* they approached a problem, not just the answer.
- Take time for the post-solution discussion. It is a very important part of the problem-solving process.
- Create a successful problem-solving atmosphere. Problem-solving ability is best improved by solving many problems in an atmosphere of exploring and sharing ideas.

SELF-ASSESSMENT QUESTIONS

for teachers of problem solving

Rate each item using the following number system
1= never 2=rarely 3= sometimes 4=often 5=always

- ☐ I ask students to explain why they chose a particular method.
- ☐ I ask students for more details after they have given a specific answer
- ☐ I ask students to give details of their reasoning process when they answer orally.
- ☐ I give students the opportunity to explore a variety of methods when solving problems.
- ☐ I allow students to explore and discover methods that work best for them.
- ☐ I demonstrate a variety of strategies for solving problems.
- ☐ I discuss with students how to choose the best strategy for. solving problems.
- ☐ I show students how to solve a single problem using more than one method.
- ☐ I explain the reasons for choosing a certain procedure or approach when discussing problems with students.
- ☐ I compare and/or contrast methods when discussing the circumstances under which a particular method is appropriate.
- ☐ I allow students to listen to the thinking processes I use when I solve problems.
- ☐ I reward students for correct methods and good ideas even when the numerical answer is not correct.

H-8

STANDARD TEXTBOOK PROBLEMS
(Word/Story Problems)

Characteristics:

- Follow development of an arithmetic operation or other algorithm
- Can be solved by direct application of one or more previously learned algorithms
- Task is to identify which operation(s) or algorithm(s) to use
- Emphasis on getting the correct answer

Purposes:

- Improve the recall of basic facts
- Strengthen skills with the fundamental operations and algorithms
- Reinforce the relationship between the operations and their applications in real-world situations

Improving Students' and Teachers' Attitudes Toward Story Problems:

Many teachers do not feel very successful in teaching story problems and many students find story problems one of the more difficult challenges in mathematics and, therefore, do not like them. Success leads to positive attitudes, and so we must begin with success. Throw out any consideration of grade level and start with very easy problems. Display or write simple problems that all the children in your group can solve.

Since long sets can be distasteful, give short sets — two, three, or perhaps four problems are plenty! However, give these sets of exercises three or four times each week.

The treatment for changing students' attitudes from negative to positive is to provide very frequent short sets of problems on which the students experience absolute success.

And what about you, the teacher? Who will help you with your attitude toward story problems? Don't worry about it. As you see your students succeeding in solving story problems and perhaps even liking them, your own attitude will improve; you will find it interesting and exciting to teach problem solving.

Source: *Problem Solving in School Mathematics* (NCTM, 1980 Yearbook)

NON-ROUTINE PROBLEMS

Non-routine problems are questions that cannot be answered with knowledge immediately available to the student.

Problem solving is the *process* by which the student resolves an unfamiliar situation or non-routine problem.

PROCESS PROBLEMS AS THE VEHICLE FOR PROBLEM SOLVING

Process Problems:

- Require use of strategies
- Stress theof obtaining the solution rather than the solution itself
- Frequently have more than one answer
- Are used to encourage the development and practice of problem-solving strategies
- Provide opportunity for cooperative learning
- Build student confidence in problem solving

GOOD PROBLEMS USUALLY HAVE ONE OR MORE OF THESE CHARACTERISTICS:

- Are in a context that interests children
- Cannot be solved directly with a computational algorithm (at least, not one known to the children)
- Have multiple solutions
- Require finding several preliminary answers to get the final solution

Source: *Problem Solving in School Mathematics* (NCTM 1980 Yearbook)

POLYA'S CONCEPTUAL MODEL FOR PROBLEM SOLVING

First: **Understand The Problem**

What do we know for sure?
Anything else?
What are we trying to find out?
Is there any information we don't understand?
Is there any information we don't need?
Any guesses or estimates of what the solution might be?

Second: **Devise A Plan**

Does the problem remind you of any we have solved before?
Would any of the following strategies help?

- sketch or diagram
- list, chart, or table
- concrete or manipulative aids
- trial and error, guess and check
- looking for patterns
- simpler numbers
- simulation or experimentation, act it out

Can we determine what operation(s) we might use?

Third: **Carry Out The Plan**

(Do not overemphasize this step. Emphasize the process of problem solving and different ways of solving the problem.)

Use the plan you devised.

Fourth: **Look Back/Extend**

(Taking the time to look back is very important but often overlooked, especially when "getting the answer at any cost" is the approach.)

Is your answer reasonable?
Is it close to your estimate?
Does it answer the question as stated in the problem?
Did anyone solve the problem another way?
Could there be other answers?
Extend the problem:

- ask "what if..." questions
- make up other problems similar to this one

USING POLYA'S MODEL

"The Dinner Party Problem"

Next month I am hosting a dinner party for thirty (30) people. I want to borrow only enough card tables, the size that seat one person on each side, to arrange them in a long row end-to-end and still seat all my guests. What is the fewest number of tables I should borrow?

Understanding the Problem

What do we know?

What are we trying to find out?

Devising a Plan

How could we go about solving this problem?

What kind(s) of sketch(s) would help us?

How could a partial sketch aid us?

Carrying out the Plan

What does our sketch(s) look like?

How many tables did we need in our solution(s)?

Looking Back

How can we be sure our solutions are accurate?

In what other way could we have solved the problem?

What if more people were invited? 42? 100? Fewer?

What if the tables were arranged differently?
In rows? In a U-shape? In an L? In a square?

VISITING FRIENDS . . .

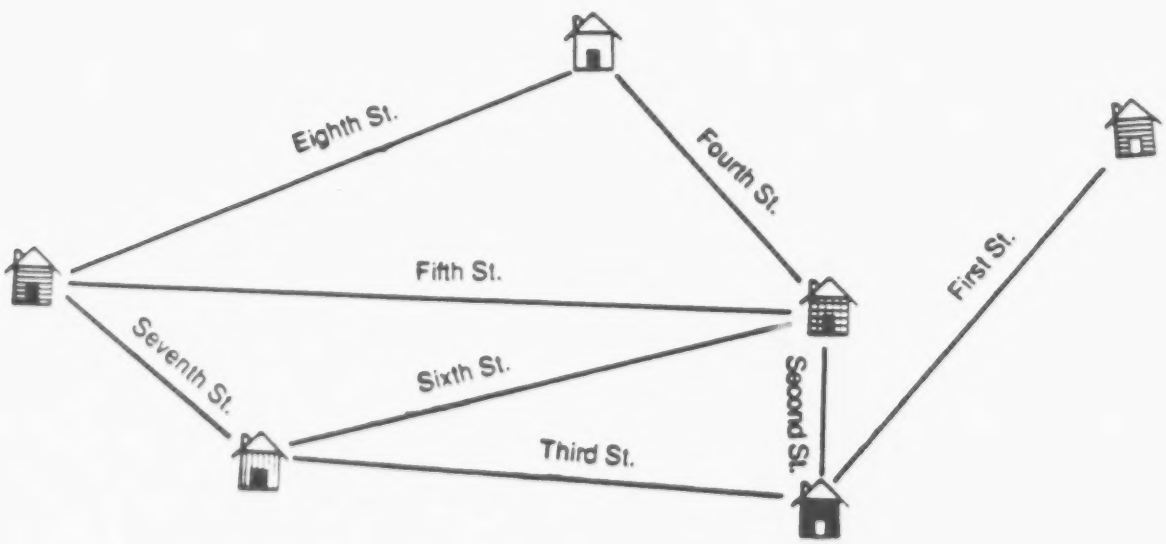
Five of my friends and I live in the same neighborhood, which has a lot of short streets.

All of us have strict parents who only allow us to visit friends who live on the same street as us. Yesterday we all sat together in the school cafeteria and figured out who could visit who.

Beth and Carol can visit.
Eva and Faye can visit.
Deb and Eva can visit.
Beth and Deb can visit.
Deb and Faye can visit.
Carol and Deb can visit.
Carol and Faye can visit.
Ann and Beth can visit.

Place each of us at our correct home.

First St.	Second St.	Third St.	Fourth St.
Fifth St.	Sixth St.	Seventh St.	Eighth St.



H-13

GETTING STUDENTS INVOLVED
IN PROBLEM DEVELOPMENT

Problems without numbers

Bon Jovi's latest release is available in both cassette tape and LP. How much could I save by buying the album rather than the cassette?

Questions

Which costs more?
What would you need to know to solve this problem?
How would you solve the problem?

Activities

Students put in their own prices and solve the problem.

Extend the problem by adding information such as, "I have a $20 bill..." or "Why do prices vary by label and artist?

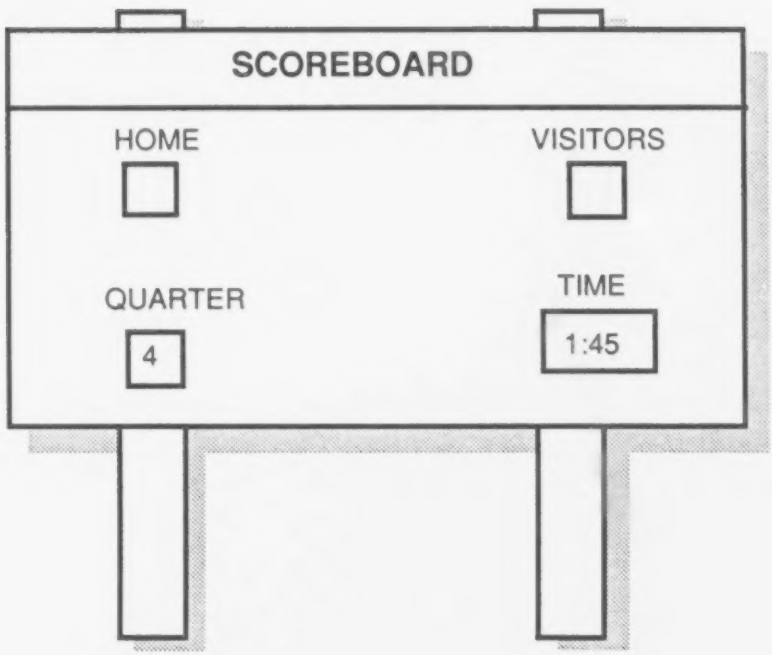
Problems without questions

Beverly and Sheila go to the store. Beverly has $5.80 and lucky Sheila has $6.20. Their intent is to buy a frisbee that costs $9.20.

Questions

Do the two have enough money?
If Sheila uses all her money, how much will Beverly pay?
What if Beverly uses all her money?
If each pays the same amount, how much would each pay?
Could they buy two frisbees?
How much more money would they need to buy two?

GETTING STUDENTS INVOLVED
IN PROBLEM DEVELOPMENT



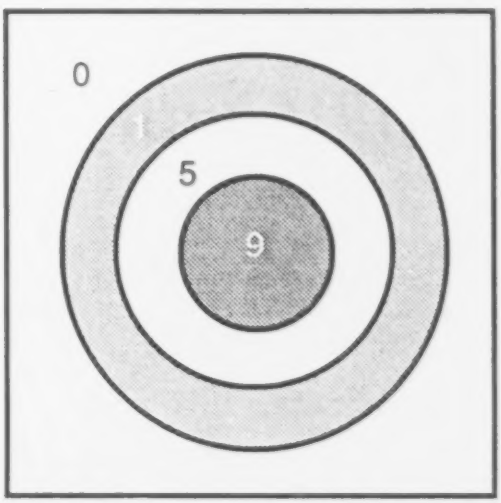
Questions

Is the game over?
How many total points were scored?
If a quarter is 8 minutes long, how many minutes have been played in the game so far?

Adapted from: "Problem Solving in School Mathematics," *NCTM 1980 Yearbook.*

TARGET PROBLEMS

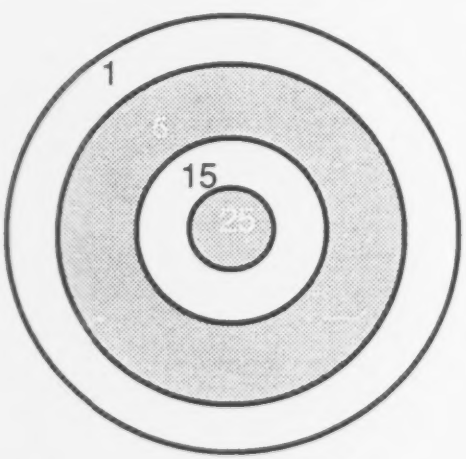
Both problems can be adapted by changing the numbers of darts thrown, point values, complexity of the target, and questions asked.



If you throw two darts, what are all of your possible scores?

If 4 darts are thrown and all hit the target, which of these scores are possible?

100	28	2
17	36	93



Adapted from: *Jesse A. Rudnick, presentation, Minneapolis, 1987.*

EASILY ADAPTED PROBLEMS

I have 5 pencils and want to give them to Chris, Joe, and Willie. How many ways can I give out the pencils if each boy gets at least one pencil?

There are 7 people at a party. If each person shakes hands with everyone else, how many handshakes are there in all?

There were 10 handshakes performed at a party. You know each person shook everyone else's hand once. How many people were at the party?

At Tom's school there are 6 basketball teams. They want to plan an afternoon tournament so that each team will play every other team once. How many games must be played?

Some children are seated at a large table. They pass around a box of candy containing 25 pieces. Ted takes the first piece. Each child takes one piece of candy as the box is passed around. Ted also gets the last piece of candy, and he may have more than the first and last pieces. How many children could be seated around the table?

Source: *Problem Solving in School Mathematics* (NCTM, 1980 Yearbook)

A farmer has some pigs and some chickens. He sent his son and daughter to count how many of each he had. "I counted 70 heads," said his son. "And I counted 200 legs," said his daughter. How many pigs and how many chickens does the farmer actually have? (For younger children, use smaller numbers, perhaps 6 heads and 16 legs.)

Last night I watched a little league baseball game. I noticed some boys and dogs playing in the grass. I heard a noise and turned to see the boys and dogs running past me. I decided to count them in a different way. I counted the legs and found there were 20. Now, what I want to know is how many boys and how many dogs ran past me?

This problem has a number of solutions. In *looking back*, you may want to add a condition such as "I counted 4 tails," or "I counted 8 heads." This will make a single solution.

Source: Adapted from a presentation by Jesse A. Rudnick, Temple University, 1987.

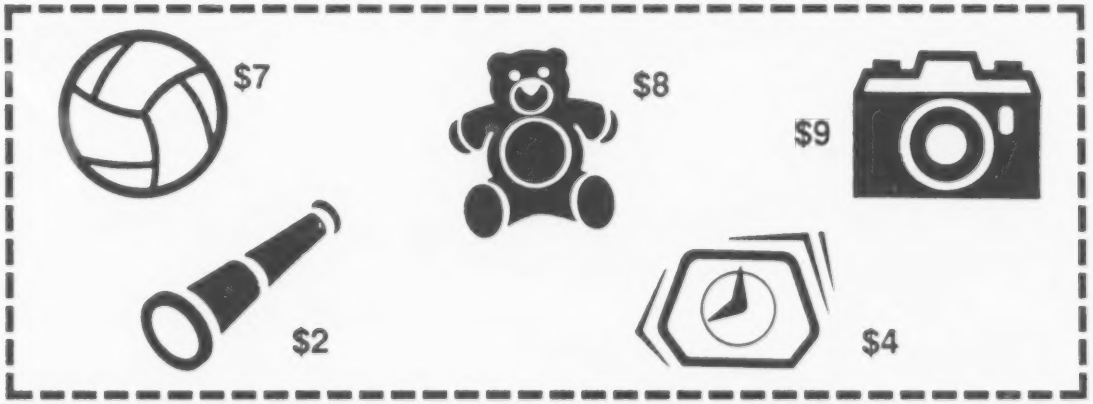
MONEY

I have nickels and dimes in my pocket that total 30 cents.
I have have 4 coins in all.
How many nickels and how many dimes do I have?

WHAT IF . . .

I had 5 coins?

I had 7 coins totaling 60 cents
and
there are nickels, dimes and quarters??



What is the total cost of the five items?
What 2 items could you buy for $13?
What 3 items could you buy for $13?
Could you buy 4 items for *exactly* $13?
If you spend $15, what could you buy?

EXTEND the problem - - -

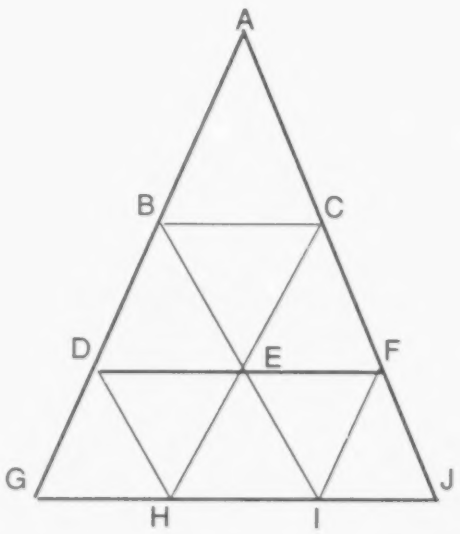
Students **develop** their own problems from the picture
Students develop **original** problems and illustrations.
Share either in small group or class-wide.

Adapted from: *John F. Firkins, presentation, Gonzaga University, 1987.*

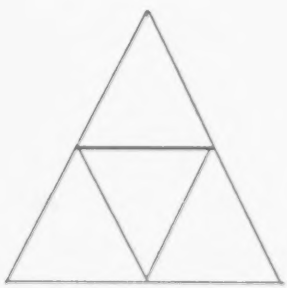
TRIANGLE

ADDITION

Place the digits **1** through **9** in the small triangles so that the sums of the four digits in the three medium triangles ADF, BGI, and CHJ are the same.



small



medium

Possible sums:

17	19	20
21	23	

Adapted from: *Mathematics Teacher*, *May 1989*.

FIND + THE + AVERAGE

Five students have test scores of **62**, **75** , **80**, **86**, and **92**.

Find the average.

How much is the average score increased IF

each student's score is increased by --

1 point 5 points 8 points *n* points

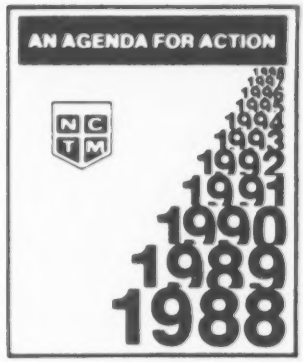
How much is the average score increased IF each individual score is increased *n* points?

Convince another student that the statement is true.

Note: Some students may need more than three trials to make a generalization. Using a calculator or computer to explore various averages and their changes will allow students to use more or greater numbers.

Adapted from: *Curriculum and Evaluation Standards, NCTM, 1989.*

Problem Solving
Tips for Teachers



Strategy Spotlight

Organizing the Classroom for Problem Solving

A growing body of research points to the benefits of having students learn in small cooperative groups. When students work in cooperative groups, the active participation of each student is maximized. More students have the chance to speak than in whole-class discussions, resulting in more opportunities for students to clarify their thinking. Also, many students feel more comfortable in small-group settings and are therefore more willing to explain their ideas, speculate, question, and respond to the ideas of others. In small cooperative groups, students' opportunities to learn with understanding are supported and enhanced.

Choosing Problems Suitable for Cooperative Groups

1. Choose problems for which a collaborative effort will benefit students, both in sharing ideas and in accomplishing the task.

2. Select problems that allow for different approaches.

3. Be sure students understand both the problem and how they are to present their results.

4. When appropriate, have students post their findings for the class.

5. Allow for discussion time to summarize, during which groups present their findings and respond to other groups' results.

An Early Grade Example—"Making Change"

Young children need opportunities for continued practice both with basic facts and with money. "Making change" is a problem-solving activity that offers both kinds of practice and also lends itself to cooperative group work.

Each group of students works to find all the ways they could be given $0.50. For example, they could receive two quarters, or one quarter and five nickels, and so on. They make a group record of their findings to present to the class. During a follow-up discussion, they also need to tell why they think they've found all the ways.

An Example for Older Children—"Number Bracelets"

"Number bracelets" appeared in the May 1980 issue of the *Arithmetic Teacher*. In this activity, students look for patterns while practicing basic addition facts. Several benefits emerge as students tackle this problem in groups. They share the task, thus making the

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work manageable, and they are able to check their conjectures and evidence with each other.

Students start by writing any two numbers from 0 to 9 and applying the following procedure: Add the two numbers and record just the digit that appears in the ones place in the sum. For example, if you start with "8 9," then the number that comes next is "7." Then add the last two numbers, the "9" and the "7," and record "6." Continue in this way, "8 9 7 6 3 9 2 . . . ," until the pattern begins to repeat.

Each group of students is asked to answer the following questions:

1. How many different possible pairs of numbers can you use to start?
2. What is the shortest bracelet you can find?
3. What is the longest bracelet?
4. Investigate the odd-even patterns in all your bracelets.
5. Make up one more question and try to answer it.

Tip Board

Guidelines for Cooperative Group Members

Three rules are useful when students work in cooperative groups. The rules need to be explained to the students and discussed, as they are only as useful as they are understood and practiced.

1. You are responsible for your own work and behavior.
2. You must be willing to help any group member who asks.
3. You may ask the teacher for help only when everyone in your group has the same question.

The third rule often puts the greatest demand on teachers when first implementing cooperative groups. Children typically ask for individual help. Asking children to check with their group, rather than giving them help at that time, is not a usual teacher response. However, it is an invaluable response for encouraging students to become more independent and to rely on each other. Assure students that you will come and discuss whatever problem the entire group faces.

How to Group Students

It is important for students to be willing to work and learn with all their classmates. Grouping students randomly accomplishes this objective.

- Students can move their desks into clusters of four each. The teacher labels each cluster with the number of a playing card—ace, two, three, and so on. The corresponding cards are shuffled and distributed; children who hold aces go to the ace cluster, children with twos to the cluster labeled two, and so forth.
- Numbered slips of paper can be drawn from a "hat" to determine random groupings of either three or four.

A Caution

Seating students in small groups does not magically produce instantly successful cooperative group work. Practice, encouragement, and discussion are required, but it is well worth the effort.

How Large Should Groups Be?

A cooperative group requires no magic number of children to work. At some times, students work best in pairs, although groups of three to six students are successful in other situations. What is important is that groups be small enough for all students to participate.

Reference

Trotter, Terrel, Jr., and Mark D. Myers. "Number Bracelets: A Study in Patterns." *Arithmetic Teacher* 27 (May 1980):14–17.

Part of the Tip Board is reserved for techniques that you've found useful in teaching problem solving in your class. Send your ideas to the editor of the section. ■

Desired Outcome for Problem-Solving

Definition

A desired outcome is a goal statement or measurable objective which focuses on what children will learn and accomplish as a result of their participation in the Chapter 1 program.

Goals

* Demonstrate a willingness to engage in problem solving.
* Be able to identify the important information in a problem statement.
* Be able to use the following strategies:

  choose the operation,
  draw a picture,
  use objects,
  guess and check, and
  look for a pattern.

Desired Outcome

80% of Chapter 1 mathematics students will show continuous growth in the three problem-solving goal areas as measured by a Problem-Solving Performance Rating Scale completed during September, January, and May.

Sample Rating Scales

Scales and their uses (as well as other methods of evaluating more advanced mathematical skills) can be found in *How to Evaluate Progress in Problem Solving* (1987) available from:

National Council of Teachers
of Mathematics (NCTM)
1906 Association Drive
Reston, Virginia 22091
(703) 620 - 9840

Student: Date:

	frequently	sometimes	never
Shows a willingness to try problems			
Demonstrates self-confidence			
Selects all important information			
Uses strategies appropriately:			
choose the operation			
draw a picture			
look for a pattern			
guess and check			
uses objects			

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END

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